Feature Article—Portfolio Constraints and Information Ratio

Active Management and Portfolio Constraints

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This article attempts to simulate and measure the impacts/effects on active management of various constraints, such as limit on the number of issues held in a portfolio, restriction of short-sales, or upper/lower limits of holding weights. The simulation results indicate that it is effective to hold many issues, as in enhanced index (EI) funds, in order to increase the efficiency of portfolio/fund operations. Constraints on holding weights, such as the upper/lower limits of weights, have a greater possibility to improve the efficiency of portfolio operations rather than lowering the information ratio, because they alleviate the negative impact on performance of extreme estimated returns. Furthermore, in some cases, such as existing active funds, the high efficiency of fund operations cannot be maintained when the number of holdings is limited, unless greater risks are taken.

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1. Emerging Enhanced Index Funds

In reviewing "manager structure," quantitative performance with respect to both past and expected future returns is highly evaluated as well as qualitative considerations such as philosophy or investment style. In this context, the ratio between active returns and active risk against a benchmark, namely the "Information Ratios" (IR), is used, and some managers have adopted "Enhanced Index (EI) funds" which restrains active risks in order to maintain IR at a high level by investing in many issues.

It is said that IR indicates the fund operational ability of fund managers. If ability is irrelevant to active risk (TE: Tracking Error) level and constant, greater tracking error means higher active returns. When TE increases, however, returns do not actually improve in proportion to TE due to various constraints and effects caused by costs, resulting in lowering IR. Thus, managers tend to avoid taking high risks in other to maintain IR. The purpose of this article is to verify the impact on IR by constraints associated with portfolio construction.

We assume active equity operations set against particular benchmarks. In constructing a portfolio, stocks are selected and active positions taken. Then decisions regarding investment weights for active stock issues are made. Of course, in some cases these processes are pursued simultaneously.

EI Funds are generally said to be these which restrain active risks by maintaining investment weights almost unchanged from those of their benchmark levels. In other words, this is a strategy where effective returns are pursued with risks restrained by narrowing the range of bets and increasing the frequency of bets simultaneously.

Grinold and Kahn (1999) showed that the following relation exists between the number of invested issues and operational ability (IR).

$$\mathrm{IR} = \mathrm{IC} \times \sqrt{\mathrm{N}} \qquad (1)$$

Here, IC (Information Coefficient) is the correlation between the (ex-post) realized active returns[1] and the (ex ante) expected active returns estimated in previous (alpha)[2]. The "N" independent expected alpha frequency per year, namely the number of all issues. In short, IR increases in proportion to alpha accuracy and the square root of the number of stocks held. (Alpha here refers to what extent the expectation/projection was achieved.)

Furthermore, Clarke, de Silva and Thorley (2002) showed that the following relation would exist if the covariance structure of active returns were not considered.

$$\mathrm{IR} = \mathrm{TC} \times \mathrm{IC} \times \sqrt{\mathrm{N}} \qquad (2)$$

Here, TC (Transfer Coefficient) is the correlation between each issue's active weight and alpha. Generally, even though managers can predict alpha with some accuracy, they cannot construct a portfolio based on such prediction. Due to fund operation constraints, they may not be able to utilize their predictions to the full. As a result, their operational ability depends not only on the number of issues held and alpha projection accuracy, but also their ability to construct portfolios, as shown in equation (1).

Here, we assume that the alpha (or structure of active risk) of each of the issues held for EI funds is identical to that of a known active fund, which means they have the same IC. The IR difference between these funds is derived from N (number of issues held) shown in equation (2) and TC (portfolio construction ability)[3].

Under the condition of long-only constraints without any short-sales, ordinary active funds focus investment on a small number of issues which carry big alphas. This lowers N as well as restrains taking issues with small (or negative) alphas for short positions. At the same time, the relevant weight becomes negative against the benchmark regardless of alpha size by holding not many issues.

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[1] *Hereafter "realized active return" is termed "active return" unless otherwise specified.*

[2] *Hereafter "active return" is called "* α *" to clearly distinguish it from realized active return.*

[3] *The question how many issues should be held depends on constraints on portfolio construction, research costs needed to increase* α *accuracy, and transaction costs of actual trading. As analysis taking these factors into account would make discussion in this article extremely complex, we do not discuss the matter here. See Section 5.*

This lowers TC. In short, EI operations enlarge N by investing smaller amounts in many issues more broadly, while increasing IR by preventing TC from dropping. Recently, another operational style, Long-Short (LS) funds has appeared, which increases TC by reducing constraints on short-sales.

Grinold and Kahn (2000)'s simulation indicated that not a negligible impact is inflicted on performance due to inefficiency deriving from holding weight constraints. Contrarily however, Frost and Savarino (1988) pointed out the possibility that the effect of estimated errors (discrepancy) in risk structure is neutralized by non-negative constraints on portfolio construction or by the upper limit of the number of individual issues held in a portfolio. Similarly, Jagannathan and Ma (2002) theoretically showed that non-negative constraints or the upper limit weaken the variance-covariance structure of total returns, resulting in positive effects which offset the estimated error of risk structure.

Based on these points, this article projects the effects of these constraints from the viewpoint of numerical analysis through simulations. Here, we also consider the concept of returns estimated in previous and actual returns in order to measure actual performance of portfolios constructed based on estimates/projections made in previous, which idea was not considered by Grinold and Kahn. Furthermore, we also verified the case where these is an upper limit on the number of issues held in a portfolio by covering not all the issues of the relevant benchmark as the investment universe. Also, simulations are made to measure the effects of the points raised by Frost and Savarino (1988) and Jagannathan and Ma (2002). As a result, the effects of constraints limiting issues held become extremely significant for operational efficiency. The possibility is shown that higher active risks can achieve higher efficiencies when the number of issues held is limited. It is also shown that constraints on weights, such as upper/lower limits, can improve operational efficiency.

This article is constructed as follows. First in the next section, effects of practical constraints are considered using a model. In Section 3, we explain the simulation framework used to project the effects stemming from the constraints. Section 4 shows the projection results. Finally, in Section 5, we give our conclusions as well as problems and possibilities not considered by the simulation.

2. Effects of Constraints on Portfolio Construction

2.1 Effects of non-negative constraints

The number of issues held by ordinary domestic active funds is approximately 100, while the number of issues comprising TOPIX (the benchmark) is as many as 1,500[4]. This means that ordinary active funds always underweight most issues according to the benchmark weights. It seems that usually they do not have a very bearish view (which means a negative alpha) regarding all the issues compared to the benchmark. On the other hand, even if they are not very bullish (positive alpha), the funds must hold issues which have high benchmark weights and high carrying risks in order to restrain TE to some degree. Considering all this, it is possible that the portfolio construction abilities (TC) of many existing active funds are fairly low, as pointed out by Clarke, de Silva and Thorley (2002).

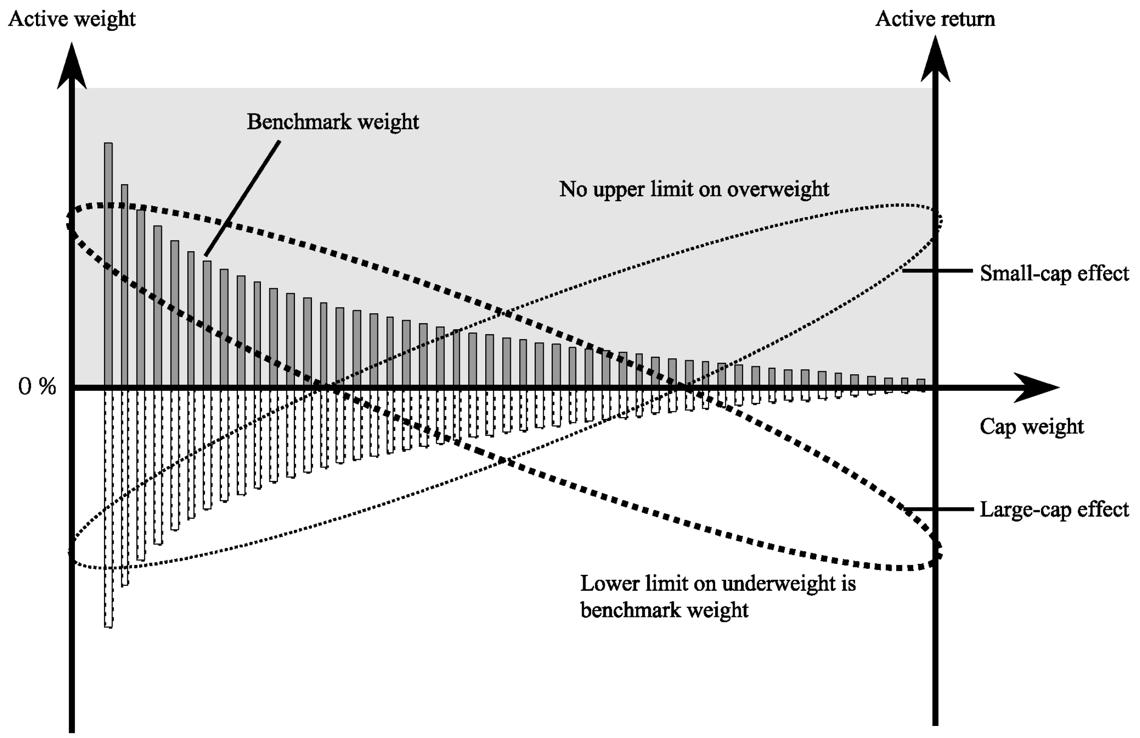
Furthermore, markets change dynamically. If one expects a market environment where returns on small-cap stocks tend to be higher in a particular period (small-cap effect), one can underweight large-cap stocks and overweight small-cap stocks so that the TC would be increased. In this case, even under non-negative constraints, large-cap stocks could be rather significantly underweighted down to their benchmark weights, while no upper limit exists on the overweight range of small-cap stocks. It is relatively easy to increase TC if the market impact is ignored. On the contrary, if one expects a market environment where returns on large-cap stocks tend to be higher in a particular period (large-cap effect), one needs to overweight large-cap stocks and underweight small-cap stocks to increase TC. Under non-negative constraints, however, small-cap stocks can be underweighted only down to a very marginal benchmark weight. Thus, large-cap stocks cannot be much overweighted and it is not as easy as in

[4] *As of the time of writing this article.*

small-cap stock markets to improve TC (see Figure 1)[5].

Flood and Ramachandran (2000) made a positive analysis of the effects from these constraints. According to the analysis, many active funds fell behind the benchmark in terms of returns in the late 1990s when the US market rose mainly due to large-cap stocks. On the contrary, it is also proven that many active funds outperformed the benchmark in the early 1990s.

Figure 1 Conceptual Diagram of Active Weight Possible under Non-Negative Constraints



2.2 Analysis using a model

Now, assume that the benchmark is a market-value-weighted stock index. N is the number of issues comprising the index. Vector $r \in R^N$ is realized active return. ω_r^2 is cross section variance. Vector $\alpha \in R^N$ is the α value estimated by fund managers. ω_α^2 is cross section variance. The active return/alpha of the i^{th} issue are r_i and α_i ($i = 1, \ldots, N$) respectively.

Also, the define $\Omega \in R^{N \times N}$ as the variance-covariance matrix of α. To simplify the matter, assume that the manager's IC is identical to all the issues comprising of the benchmark. Thus, the IC is the correlation between r and α at the cross section.

$$IC = \frac{Cov[r, \alpha]}{\omega_r \ \omega_\alpha}$$

The number of issues held in a portfolio of a manager is represented by n. Vector $x \in R^N$ is the active weight of each issue of

[5] *If TC does not change in accordance with market changes, the following could be the explanation. As large-cap stock issues have more influence on performance, more research costs are spent on large-cap issues. As a result, values are more accurately estimated for the α (or risk structure) of large-cap issues. In other words, actual IC is not common among all issues. Larger cap issues may have higher ICs, while smaller cap issues may have smaller ICs. In fact, positive effects from the relatively high α accuracy of large-cap issues offset negative effects from the non-negative constraints on portfolio construction. The stronger side of effects may appear.*

the portfolio. The x_i is the active weight of the i^{th} issue. σ_x^2 is cross-section variance. The manager's TC is the correlation between α and x at the cross section, shown as follows. Vector $b \in R^N$ is the benchmark weight of individual issues. Scholar b_i is the benchmark weight of i^{th} issue.

$$TC = \frac{Cov[\alpha, x]}{\omega_\alpha \ \sigma_x}$$

Now, a manager structures a portfolio minimizing active risks to achieve a target expected value "t" for the investment return which outperforms the benchmark performance required to be achieved by investors. Practically, fund constraints are set to make the sum of active weights zero (0) so as not to cause a shortage/excess of funds. Thus, the active weight of respective issues is x which satisfies the constraints of the following variance minimization question.

$$\begin{aligned} \text{Min.} \quad & \frac{1}{2} x^T \Omega x, \\ \text{s. t.} \quad & \alpha^T x = t, \\ & e^T x = 0. \end{aligned} \tag{3}$$

(Here, ^{T} represents the transposed matrix. $e^T = [1 \ldots 1] \in R^N$ and t is the target return.)

More realistically, the existing active operation style selects one "n^{th}" issue for investment among all N issues comprising of the benchmark. The following is the variance minimization question where a constraint requires the other issues not to be held[6].

$$\begin{aligned} \text{Min.} \quad & \frac{1}{2} x^T \Omega x, \\ \text{s. t} \quad & \alpha^T x = t, \\ & e^T x = 0, \\ & diag[-(h-1)]x = -diag[-(h-1)] \ b. \end{aligned} \tag{4}$$

(Here, $h \in R^N$ is a vector of holdings that shows the issue held is 1 and that the issue not held is 0. diag [・] is the operator which diagonalizes the vector.)

One can also use the following equation setting the constraint. This constraint requires portfolios to hold issues at the same weights as benchmark weights. This is different from just "not investing in issues which are excluded from the investment universe."

$$\begin{aligned} \text{Min.} \quad & \frac{1}{2} x^T \Omega x, \\ \text{s. t.} \quad & \alpha^T x = t, \\ & e^T x = 0, \\ & diag[-(h-1)]x = diag[-(h-1)] \ O. \end{aligned} \tag{4'}$$

(Here, $O \in R^N$ is the vector which shows all the elements are 0.)

Furthermore, it is more common to set the constraint on holding weights of respective individual issues for investment as follows, in

[6] *This is so-called "long/short strategy".*

addition to equations (4) and (4')[7].

$$
\begin{aligned}
\text{Min.}\quad & \frac{1}{2}x^{T}\,\Omega\,x,\\
\text{s. t.}\quad & \alpha^{T}x = t,\\
& e^{T}\,x = 0,\\
& \mathrm{diag}\left[-\left(h-1\right)\right]x = -\mathrm{diag}\left[-\left(h-1\right)\right]\,b,\\
& ub \geq \mathrm{diag}\left[\,h\,\right]\;x \geq -\,\mathrm{diag}\left[\,h\,\right]\,b.
\end{aligned}
\tag{5}
$$

(Here is the vector $ub \in R^{N}$ which indicates the upper limit of active weight of respective issues.)
Or it would be more common to put the variance minimization question as follows[8].[9]

$$
\begin{aligned}
\text{Min.}\quad & \frac{1}{2}x^{T}\,\Omega\,x,\\
\text{s. t.}\quad & \alpha^{T}x = t,\\
& e^{T}\,x = 0,\\
& \mathrm{diag}\left[-\left(h-1\right)\right]x = \mathrm{diag}\left[-\left(h-1\right)\right]\,O,\\
& ub \geq \mathrm{diag}\left[\,h\,\right]\;x \geq -\,\mathrm{diag}\left[\,h\,\right]\,b.
\end{aligned}
\tag{5'}
$$

TC (portfolio construction ability), which is the correlation between α and active weight x, is significantly constrained by setting these constraints in addition to equation (3). Next, we confirm these effects with the simulation.

3. Analytical Framework

This section projects the above-mentioned effects caused by the number of issues held or non-negative constraints, or upper/lower limits of active weights on performance, simulating them as follows:

① First, generate realized actual returns based on a normal random number of N issues.

② Assume that prediction accuracy is IC for realized active returns. And, generate estimated α using the Cholesky decomposition method.

③ Based on this estimated α, construct a portfolio. Set the following constraints in accordance with equation (4) or (5).

a) Limit on number of issues held

b) Non-negative constraint

c) Upper/lower limits of active weights

④ Measure the portfolio's realized active return, active risks, and realized IR.

The above steps were repeated 1,000 times for every case of constraints and respective average figures were calculated for realized active return, active risk, and realized IR.

The number of issues comprising the benchmark is set at 500 for convenience. No consideration is made for costs related to α estimation or portfolio construction.

[7] *This is ordinary "active strategy".*

[8] *This can be the same as enhanced indexing funds.*

[9] *More practically, it is common to set constraints on sector weighting or factor exposures. To simplify discussion, these issues are not considered here.*

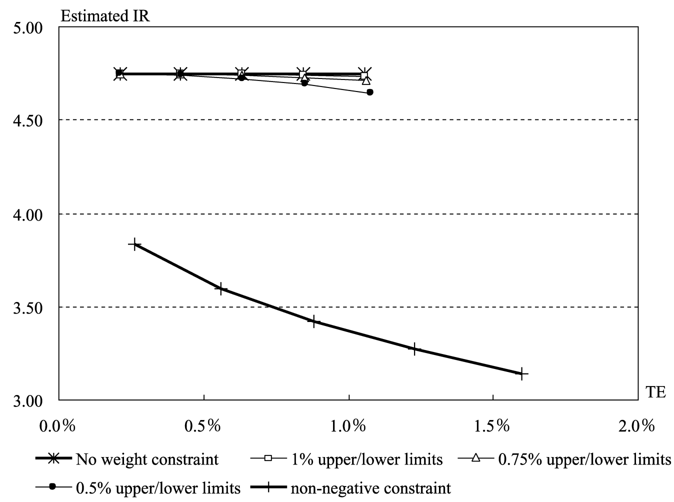
4. Results of Analysis

4.1 Effects from constraints of holding weights

To see the effects of "holding weights constraints" (non-negative constraint or upper/lower limits), compare results of equations (4) and (5) for the n=500 case (total 500 issues consisting of the benchmark). Each equation minimizes its target function, using estimated α series. Figure 2 shows de-facto performance before using the portfolio's estimated α calculated by the equation. And Figure 3 show the portfolio's de-facto performance after using the realized active return series. For comparison, we also describe the respective results when we set the upper/lower limits of active weights at 1%, 0.75%, and 0.5%[10].

First, we assess the effects of holding weight constraints on portfolio construction, using the α estimated in previous. Figure 2 shows the average value of estimated IR and TE, generated by optimizing each simulation for cases where there is no weight constraint, and that there are upper/lower limits on active weights and that there is a non-negative constraint. According to this, if there is no weight constraint at all, the higher the target active return "t" is, the higher TE is. This is a linear relation. Thus, estimated IR is constantly the same, irrelevant to the target active return level. However, if there is an upper/lower limit or non-negative constraint, one sees that higher target active return levels tend to lower IR. As assumed qualitatively to a certain degree, estimated IR under a non-negative constraint lowers rather rapidly when TE increases. This is a significant constraint in terms of portfolio construction. The upper/lower limit can be a constraint factor for portfolio construction as the possible range of active weights is narrowed[11]. The effects, however, are not as significant as those of non-negative constraints.

Figure 2 Effects on IR by Weight Constraints (1) [500 issues, estimated active returns]

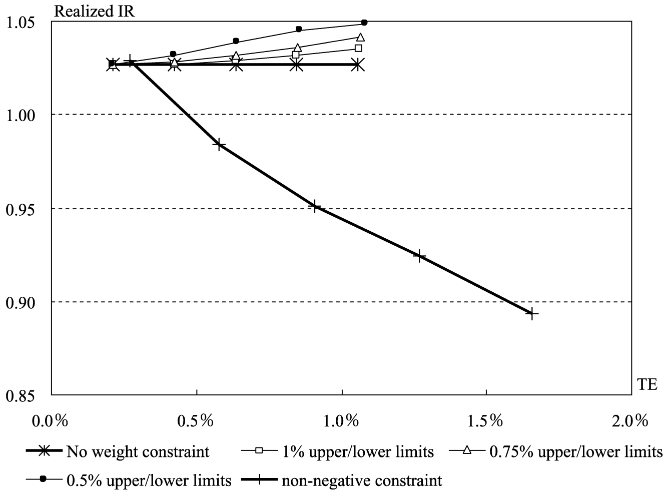


Note: Here, TE means active risks and realized IR shows α/TE.

[10] *In some cases, upper/lower limits are set at relative ratios against benchmark weights. However, here we assume constraints are set at absolute levels of active weights.*

[11] *In the simulations, the benchmark weights of respective stock issues are generated to make logarithms of the benchmark weights at normal distribution using normal random numbers, so that benchmark weights will approximate actual weights. As a result, benchmark weights here are approximately 2% at maximum and approximately 0.01% at minimum.*

Figure 3 Effects on IR by Weight Constraints (2) [500 issues, realized active returns]



Note: Here, TE means active risks and realized IR shows α/TE.

Next, we confirm such de-facto performance. Figure 3 is the result measured on a realized-return basis. The realized de-facto active return and TE where there is no weight constraint are the same as those before. The higher the target active return "t" becomes, the higher TE becomes. This is a linear relation.This simulation chose 0.05 as the IC level, which shows how much of the active return expected before the fact can be realized after the fact. Thus, although the level itself is very different from that of the estimated IR, the realized IR becomes constant regardless of the target active return level. Suppose that the relation in equation (2) exists here. Then, TC in this case would be projected as follows: [12]

$$\begin{aligned} TC &= \frac{IR}{IC \times \sqrt{N}} \\ &= \frac{1.027}{0.05 \times \sqrt{500}} \\ &= 0.919 \end{aligned}$$

On the other hand, the non-negative constraint, which is a very big restriction on estimated IR before the fact, also significantly influences after the fact. The higher the target active return is set, the more conspicuous the effects become. In the case where "t" is 5.0%, realized active return becomes 1.426% and TE becomes 1.655%. If one uses equation (2), TC is calculated to be 0.800. This means that portfolio construction ability is reduced by approximately as much as 13%, compared to the case where there is no constraint. The realized active return level, however, outperforms that in the case of no constraints. TC lowered rather due to increased active risk. This can be explained as follows. Non-negative constraints force investment to focus on issues which bring a positive α. This results in lowering risk diversification efficiency and estimated accuracy. In the case of high risk/high return, the impact becomes more significant.

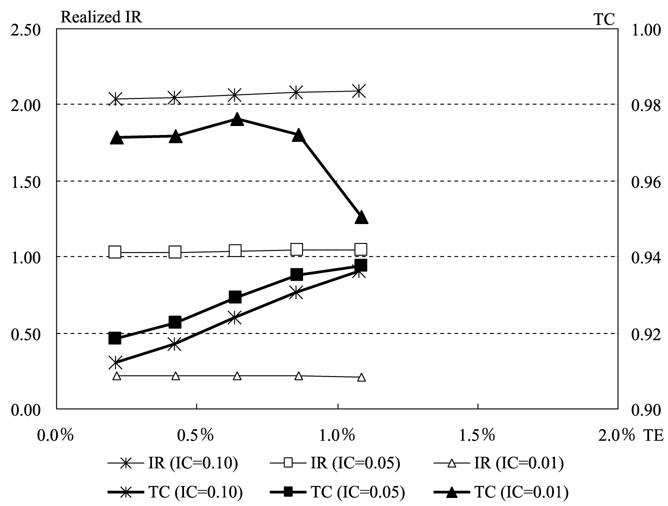
Furthermore, we obtained a very interesting result for the after the fact effect from the upper/lower limits, which were relatively marginal constraints for portfolio construction in terms of estimated IR before the fact. The narrower the range of possible active weights becomes or the higher the target active return level becomes, the higher realized IR becomes. If the upper/lower limit of active weights is 0.5%, TC (t=5.0%) is calculated to be 0.937 in equation (2). This means that portfolio construction ability improves by approximately 2% compared to the case of no constraints. These phenomena can be interpreted as follows. If no

[12] *Realized IR is measured using realized active returns carrying a correlation between the estimated α before the fact and IC. The realized IR level itself is affected by cross-section variance ω^2_r, which is used for simulation. This is probably the reason why TC becomes smaller than theoretical value, 1.*

upper/lower limit exists for issues carrying α estimated to have extremely large absolute values, extremely large active weights are set for such issues through the optimization process. If this estimated high α (active return) is not achieved, performance is very negatively influenced. However, if one set upper/lower limits, one can avoid these extreme weightings and alleviate the negative effect on performance. This can improve portfolio operational efficiency. Thus, it is observed that these upper/lower limits have the effects pointed out by Frost and Savarino (1988) and Jagannathan and Ma (2002), although they are minor.

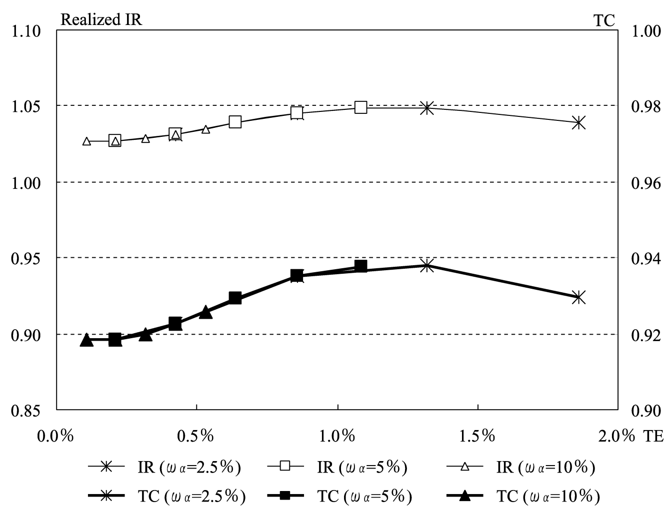
It is assumed that the positive effects these upper/lower limits have on performance depend on α accuracy (IC) and variance of cross-section distribution of estimated active returns. A higher IC has smaller effects, while a lower IC has bigger ones. The broader cross-section variance is, the stronger the effect becomes. The narrower the variance is, the weaker the effect, Figure 4 shows the effects of different ICs and Figure 5 illustrates the results of the effects caused by different cross-section variances. The upper/lower limits on weights are all set at 0.5%.

Figure 4 Effects on IR by Weight Constraints (3) [500 issues, realized active returns]



Note: Here, TE means active risks and realized IR shows r/TE. TC is calculated by equation (2). The active weight of upper/lower limits is set at 0.5%.

Figure 5 Effects on IR by Weight Constraints (4) [500 issues, realized active returns]



Note: Here, TE means active risks and realized IR shows r/TE. TC is calculated by equation (2). The active weight of upper/lower limits is set at 0.5%.

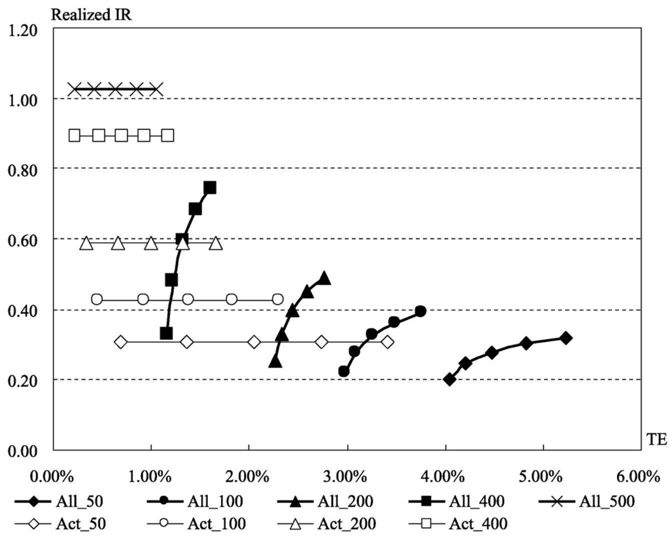
First, we discuss the effects weight constraints have on portfolio construction for different ICs. Naturally, a higher IC heightens the realized IR level after the fact. If IC is increased more and more from 0.01 through 0.05 to 0.10, the difference in realized IR levels becomes wider and wider. It is evident, however, that TC tends to be higher when IC is lower. If ICs are 0.10 and 0.05 respectively, the IR becomes higher provided the target active return “t” (or TE) is set higher. If the IC is 0.01, TC increases until TE reaches around 0.6%. However, if TE increases above this level, TC lowers. This can be explained as follows. As “t” increases, the negative effects on portfolio construction caused by upper/lower limits become bigger than the positive effects from the estimated errors in active returns.

Next, we see how portfolio construction is influenced by weight constraints derived from different cross-section variance ω^2_α of estimated active returns. Effects caused by the ω_α change depend on the target active return “t” level. It can be said that a smaller ω_α can alleviate negative effects on portfolio construction stemming from upper/lower limits on weights. On the other hand, however, an extreme position needs to be taken in order to achieve the same level “t,” compared to the higher ω_α case. Influenced by this, portfolio construction ability (TC) weakens. These factors are the reasons why ω_α starts to lower when TE reaches around 1%, while ω_α is 2.5% . On the contrary, it is assumed that when ω_α is 10%, TC improvement remains at a relatively low level, as the constraint from “t” is relatively moderate.

4.2 Effects from different number of issues held in portfolios

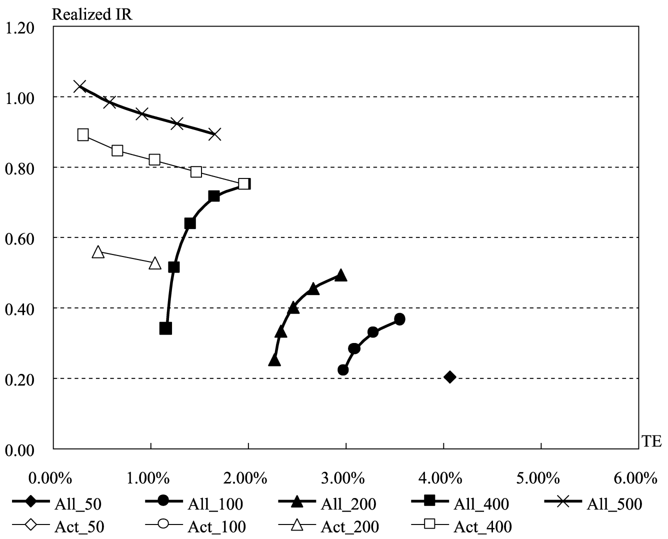
Next, we discuss how active ability is influenced by the number of issues held. As is the case where there is no weight constraint on issues included in portfolios, Figure 6 shows the relation between realized IR and active risks when “n” (number of issues in the investment universe) is changed in equation (4). In addition, another case is illustrated to compare the effects made by non-held issues. It transpires that issues not to be held, are held at benchmark weights, while other issues are actively held. This is the result of equation (4’). Figure 7 shows similar results from equations (5) and (5’) where there are non-negative constraints and a 5% upper limit.

Figure 6 Effects on IR by Constraints of Issues Held (1) [Realized active returns with no weight constraint]



Note: All_50, All_100, All_200, All_400, and All_500 are cases where the shown number of issues are invested in by the portfolios. Act_50, Act_100, Act_200, and Act_400 are cases where all issues are held in portfolios which actively hold 50 issues, 100 issues, 200 issues, and 400 issues, respectively, while the rest are held at the benchmark weight. And, TE means active risk and realized IR shows r/TE.

Figure 7 Effects on IR by the Constraint of Issues Held (2) [Realized active returns with weight constraint(non-negative and 5% upper limit)]



Note: All_50, All_100, All_200, All_400, and All_500 are cases where the shown number of issues are invested in by the portfolios. Act_50, Act_100, Act_200, and Act_400 are cases where all issues are held in portfolios which actively hold 50 issues, 100 issues, 200 issues, and 400 issues, respectively, while the rest are held at the benchmark weight. And, TE means active risk and realized IR shows r/TE.

Now, we discuss the different effects occasioned by the number of issues held where there is no weight constraint (Figure 6). All_50, All_100, All_200, All_400, and All_500 in the following diagrams are cases of investing in the shown number of issues. Act_50, Act_100, Act_200, and Act_400 are cases where all the issues are held, while 50, 100, 200, and 400 issues are held on an active-operation basis, and other issues at benchmark weights. When the number of issues held in a portfolio is 500, realized IR is constant, irrelevant to target active return "t" and TE. This is same as where there is no weight constraint as described in the former section. Also, the realized IR level is constant, irrelevant to "t" and TE, although the realized IR level itself lowers where a certain portion of the issues are held at the active weight and the rest at benchmark weights. Of course, the realized IR level becomes low. This is because if a smaller number of issues are held actively, the constraint becomes more strict.

The tendency is quite different from where a portfolio does not include all the issues consisting of the benchmark. The result: the higher "t" and TE become, the higher realized IR becomes. Here is the interpretation of the phenomenon. If the number of issues held is limited, the issues not to be held are actively underweighted against benchmark weights. To set them so as not to be held generates active returns and active risks. Such non-holding constraints account for most of the active factors when the increase in "t" relatively weakens the strength of the effect. This turned out to show the seemingly strange result where the higher "t" and TE are, the higher realized IR becomes. The realized IR level lowers because the smaller the number of issues held in a portfolio, the more significant the effect of the non-holding constraints becomes. When looked at from the viewpoint of realized active return levels, the smaller the number of issues held, the higher active returns tend to be. Thus, the effects on portfolio construction of "issues not to be held" may work to lower active operational ability, as they cannot diversify active risks very well.

Now, we discuss the case of non-negative constraints and a 5% upper limit (Figure 7). If 500 issues are held in a portfolio, the realized IR lowers if we set the target active return "t" higher. This is the same result as the analysis discussed in the previous section. Furthermore, when a certain portion of the 500 issues is held at an active weight, realized IR lowers as "t" increases. This is due to the impact of non-negative constraints and upper limits. However, when one compares All_200 and Act_200, All_200 has the higher realized IR with lower risks. This is because in the All_200 case, one cannot decrease risk due to the issues not held, resulting in high risks and decreased efficiency. In other words, if one holds "issues not decided actively" at the benchmark weight, one can decrease risk and maintain a high IR. These results can explain why EI fund operations with low risks are adopted. As Act_50, Act_100, and Act_200 cannot realize the target portfolio at certain “t” levels, such portions are omitted in the diagram.

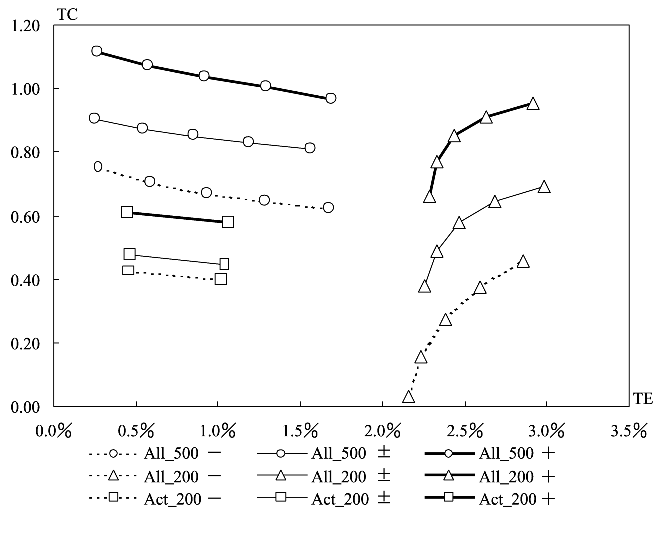
Next, if a portfolio does not hold all the issues comprising the benchmark, the higher “t” and TE become, the higher realized IR becomes. This is the same as the case of no weight constraint. The same interpretation of the case of no weight constraint can be applied to this case. If weight constraint is added to the non-holding constraint, the total constraint becomes bigger than just the non-holding constraint. Thus, even if one increases “t,” realized IR improves less moderately than in the case of no weight constraint. If the number of issues held becomes smaller, realizable portfolios which can satisfy the constraint become more limited. Thus, the unrealizable portfolios of the All_50 and All_100 cases are not shown in the diagrams. Often in actual active operations, the number of issues held is considerably limited and non-negative constraints and upper/lower limits are set for active weights. In these cases, one can say that one might not be able to sufficiently increase operational efficiency unless one increases TE substantially.

4.3 Effects of size factors

Finally, to observe the effects of non-negative constraints in more detail, we confirm the effects of non-negative constraints when there is a size-factor effect, assuming a certain market development where the (issue) size factor can explain some of the active returns. Figure 8 compares the results where the number of issues held is 200, where the number of issues held is 500 issues of which 200 are held actively, and where the number of issues held is 500 issues all of which are held actively. And, all cases here have non-negative constraints. The simulations were made 1,000 times for each case. The 1,000 simulations are categorized into the following three: 300 simulations where smaller cap stock issues have lower active returns (small-cap effect －, thin dotted line),300 simulations where smaller cap issues have higher active returns (small-cap effect ＋, heavy solid line), and 400 simulations for the rest of the cases (small-cap effect ±, thin solid line).Based on the above categories, we show the relation between TC and TE calculated by equation (2).

We can confirm with the result of the simulations that realized IR increases and portfolio construction ability (TC) rises in all cases if the small-cap effect becomes stronger. This means the following: if small-cap issues have low active returns (big negative value), large-cap issues can be overweight up to the upper limit, but small-cap issues cannot be significantly underweight due to the non-negative constraints. This results in lowering TC. On the contrary, when the small-cap effect is strong, small-cap issues can be

Figure 8 Effects by Size Factor of Non-negative Constraints [Realized active returns]



Note: Here, the All_200 and All_500 are cases investing only in the shown number of issues. Act_200 is where the portfolio holds all issues of which 200 issues are held actively and the remaining are held at the benchmark weight. TE means active risk and TC value is calculated by equation (2).

overweight up to the upper limit and large-cap issues can be underweight maximally at the relatively large benchmark weight. Thus, TC will not fall much. Thus, non-negative constraints have asymmetric effects, depending on the market situation where large-cap issues or small-cap issues rise.

5. Conclusion and Issues for the Future

In this article, we attempted to measure the effects of various constraints on portfolio construction in terms of IR, the operational ability of active operations. A similar study was made by Grinold and Kahn (2000). In this article, however, we obtained the following new insights, considering not only non-negative constraints, but also weight constraints, such as upper/lower limits, and the correlation between returns before/after the fact related in terms of IC and "issues not to be held" in portfolios among issues comprising the benchmark.

First, non-negative constraints potentially and significantly lower portfolio construction ability (TC). On the other hand, it was confirmed that upper/lower limits lowers efficiency on the frontier before they are used for portfolio construction but efficiency improves marginally on the frontier after they are used for portfolio construction. If there is no upper/lower limit, as active returns are estimated to have extremely large absolute values, sometimes extremely large active weights are placed through the optimization process. As their realized returns are not necessarily large values, performance is negatively influenced. On the other hand, as Frost and Savarino (1988) and Jagannathan and Ma (2002) pointed out, these extreme weightings can be avoided by setting upper/lower limits. It is possible that such alleviates the effects on performance and improves operational efficiency. It was also confirmed that such effects depend on α accuracy (IC) and that the smaller IC becomes, the more conspicuous the effect of improving operational efficiency by upper/lower limits becomes. It was assumed that if the variance of cross-section distribution of active returns is bigger, the effect must be stronger. The simulation results, however, show that they largely depend on the target active return levels of portfolios. In the case of small variance, upper/lower limits affect portfolio construction less negatively. One has to, however, take extreme positions to achieve the same target return "t," compared to large variance cases. It seems that these effects reduce portfolio construction ability (TC).

Next, where a portfolio does not hold all issues comprising the benchmark, one realizes higher IR if target active returns and active risks are set higher. This can be explained as follows. The constraint restricting the holding of certain issues becomes a big constraining factor when the target active return is low. However, as the target active return becomes higher, the effects weaken. Furthermore, the smaller the number of issues held becomes, the stronger the effects of non-holding constraints become. The level of realized IR lowers. On the other hand, the realized active return level tends to become higher. Thus, when there are "issues not to be held," the effects on portfolio construction can reduce active operational ability due to the insufficient distribution of active risks. On the contrary, this means that risks can be reduced and lower efficiency avoided by holding issues other than actively-held ones at the benchmark weight. This provides the ground for EI operations.

We also analyzed the effects of non-negative constraints where size factors exist. The simulation results confirmed the following. The stronger small-cap effects, the greater portfolio construction ability (TC). And, operational efficiency tends to increase. This can be also explained as follows. In the large-cap stock market, large-cap issues can be overweight up to the upper limit, but small-cap issues cannot be much underweighted due to non-negative constraints. This lowers TC. On the contrary, in a stock market where small-cap effects are strong, small-cap issues can be overweight up to the upper limit and large-cap issues underweight maximally at the big benchmark weight. Thus, TC has little room to fall. Therefore, non-negative constraints can have asymmetrical effects, depending on market developments, such as on markets for large-cap stocks and small-cap stocks.

The above results are the conclusion derived from the simulation set within the assumed cases. Here, we should point out the possibility that another conclusion could be deduced from assuming various other factors not considered in this article. First, in this analysis, costs are not considered at all. Specific fund sizes are not assumed, either. In particular, if funds are huge, enormous costs may be incurred. In this case, the effects at the size factor would be different. In other words, it is assumed that TC in the small-cap stock market may not outperform TC in the large-cap stock market. Furthermore, given the research costs needed to generate α, it can be a non-efficient task to estimate α of thinly-traded small-cap issues with enormous market impact at the same accuracy as that of other issues. In terms of the effects of the constraint on number of issues held, when the number of issues held is increased, the cost associated with rebalancing the fund cannot be ignored. To avoid unnecessary complexity in this article, our analysis did not consider these effects and they remain to be addressed in the future.

Finally, the simulation in this article indicated several useful possibilities. However, our discussions were for cases using estimated values, such as α and risk which had already been used in existing operations. Most important for portfolio operations is to improve operational efficiency by increasing the number of issues to be researched and improving α accuracy, not to mention making efforts to improve operational efficiency.

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