Security Analysts Journal
Vol.43 No.9
September 2005

Security Analysts Journal Prize 2005

Factors Driving Correlations Between Fixed Income and Equity Returns

– Asset Allocation and ALM for Pension Funds –

Junichi Iwamoto CMA
Chief Researcher
Pension Research Institute
The Sumitomo Trust & Banking Co., Ltd.

Contents

Abstract

Asset allocation requires estimating not only returns but also risks. In practice, the volatilities of assets returns or the correlation coefficients between them is commonly estimated by historical data. However, Ilmanen (2003) and Asano (2005) have pointed out that the correlation coefficient between fixed income and equity returns have recently exhibited a significant shift. It is difficult to identify important factors for the correlation coefficient intuitively because the correlation coefficient indicates a relative relationship between fixed income and equity. This article does not focus on the correlation coefficient itself. Rather, it alternatively employs a method to estimate the correlation coefficient by decomposing expected returns of equity and fixed income into several factors. The result of our empirical study proves that the most important factors are real interest rates and expected dividends.

Junichi Iwamoto, CMA: Graduated from Keio University, in 1987 and joined The Sumitomo Trust & Banking, and then assumed his current position in April 2005. In 1994, he graduated from the Graduate School of International Management, International University of Japan and in 2004 the Graduate School of International Corporate Strategy (ICS), Hitotsubashi University. His writings include a paper entitled “The Decision Support Model for Corporate Bond Investment in The Sumitomo Trust” (July 19, 2000, *Finance Frontier, The Nikkei Financial Daily*) and “Fixed-Income Fund Management Style and Performance” (November 2001, *Security Analysts Journal*), and “Asymmetric Information concerning Prepayments and Risk Management of MBS Investment”（co-authored, March 2004, *Financial Review*）.

1. Estimation of Correlation Coefficients

Recently, the appropriate level of an equity risk premium or an expected return of equity is being discussed in Japan and the United States. In portfolio theory or fund management, it is evident that expected returns are the important factors for asset allocation. However, the actual estimation method is not always obvious. The application of past risk premiums to the future by the historical approach have been common among actuaries, consultants, and sponsors. However, it has been pointed out that this approach tends to exaggerate expected returns.

For example, when Arnott and Bernstein (2002) decomposed historical data of high equity returns into several factors, they found that escalation in variation levels and dividend yields explained the most of returns, and equity risk premiums explained less. They claimed that an expected equity return should reflect these results. Correspondingly, Ibbotson and Chen (2003) have suggested the supply-side approach where an equity risk premium comes from value added in the economy. Additionally, they tried to estimate an expected return by assuming that a change in the variation level in the future could be ignored. Influenced by heated debates in the United States, Suwabe (2003), Miyoshi (2003), Yamaguchi (2003) and others are actively conducting a variety of studies for an expected return of equity by employing the supply-side approach and the implied approach from an equity price in Japan.

It is obvious that asset allocation requires estimation of not only expected return but also risk. In practice, the risk of each asset or the correlation coefficients between returns has been commonly estimated based on historical data, in the same way as expected returns have been estimated. The risks (volatility) and the correlation coefficients are assumed stable when compared to expected returns. However, in recent years, researches by Ilmanen (2003) and by Asano (2005) has shown that a recent correlation coefficient between fixed income and equity returns have significantly varied from the past levels, in both Japan and the United States. These researches indicated that whilst correlation coefficients were formerly high, they have since rapidly decreased. However, no consistent discussion of the background, as has been the case for expected returns, has been conducted. Hence the background to the shift in correlation coefficients still remains unclear.

A correlation coefficient indicates a relative relationship between fixed income and equity, so it is difficult to intuitively identify what factors have potential influence on it. This article does not discuss the correlation coefficient itself. Rather, it adopts a method to first decompose expected returns of equity and fixed income into several factors, and estimates correlation coefficients. The article then clarifies some factors that consistently influence the correlation coefficient. Finally, the article employs actual data in order to clarify which factor is important.

2. Decomposition of Expected Rate of Return by the Factor Model

Before decomposing the correlation coefficient into several factors, this section shows that returns of fixed income and equity by factor models are derived from the following price formulas. In the first formula for equity price P_s, equity dividend for period t is denoted by D_t, expected nominal growth rate of a dividend by g_n, expected real growth rate by g_r and expected equity risk premium by k. In the second formula for a government bond price P_B, the cash flow of fixed income for period t is denoted by CF_t, a coupon by C, nominal interest rate by y_n, real interest rate by y_r and expected inflation rate by l.

$$P_s = E\left[\sum_{t=1}^{\infty}\frac{D_t}{(1+y_n+k)^t}\right] = E\left[\frac{D_0}{y_n+k-g_n}\right] = E\left[\frac{D_0}{y_r+k-g_r}\right] \quad (1)$$

$$P_B = \sum_{t=1}^{T}\frac{CF_t}{(1+y_n)^t} = \frac{C}{y_r+l}\left\{1-(1+y_r+l)^{-T}\right\} + 100\times(1+y_r+l)^{-T} \quad (2)$$

From equations 1 and 2, it is understood that the following factors are important for an equity price: a real interest rate, an expected risk premium, and a dividend growth rate. Simultaneously, a real interest rate and an expected inflation rate are supposed to be important factors for a fixed income price. Factor decompositions of the returns can easily be done using the above equations. These formulas approximate capital gains using Taylor expansion ignoring terms exceeding the term to the second degree. Then, adding income gains, the factor decompositions of a fixed income return and an equity return can be conducted as follows.

First, equation 3 expresses the decomposition of a TOPIX (including dividends) return. The last term of D is impliedly estimated by deducting each factor on the right-hand side from a realized equity return on the left-hand side. $\partial P_s/\partial y_r$ and $\partial P_s/\partial k$, which indicate capital gain or loss, are obtained by regression analysis without a constant term by TOPIX (without dividends) returns for the past 12 months. Section 3.1 will explain how to estimate y_r and k.

$$\begin{aligned} r_s &= d + \frac{\partial P_S}{\partial t}\Delta t + \frac{\partial P_S}{\partial y_r}\Delta y_r + \frac{\partial P_S}{\partial k}\Delta k + \frac{\partial P_S}{\partial g_r}\Delta g_r + \frac{\partial P_S}{\partial D_0}\Delta D_0 \\ &= y_r + k + l + \frac{\partial P_S}{\partial y_r}\Delta y_r + \frac{\partial P_S}{\partial k}\Delta k + \left(\frac{\partial P_S}{\partial g_r}\Delta g_r + \frac{\partial P_S}{\partial D_0}\Delta D_0\right) \\ &= y_r + k + l + \frac{\partial P_S}{\partial y_r}\Delta y_r + \frac{\partial P_S}{\partial k}\Delta k + D \end{aligned} \quad (3)[1]$$

[1] *Based on* $d = D_0/P_S = y_r + k - g_r$ *and* $\partial P_S/\partial t = g_n = g_r + l$. *In addition,* ΔD_0 *can be decomposed not into a nominal value, but into a change in real value and inflation rate. But it seems unlikely that the upcoming dividend for the next period is significantly changed due to only a change in the current inflation*

Secondly, equation 4 shows the factor decomposition of a fixed income return. Section 3.2 will give the detailed explanation of the specific estimation method for an expected inflation rate. $\partial P_B / \partial y_r$ and $\partial P_B / \partial l$ are estimated from the adjusted duration of NOMURA-BPI.

$$\begin{aligned} r_B &= c + \frac{\partial P_B}{\partial t}\Delta t + \frac{\partial P_B}{\partial y_r}\Delta y_r + \frac{\partial P_B}{\partial l}\Delta l \\ &\approx y_r + l + \frac{\partial P_B}{\partial y_r}\Delta y_r + \frac{\partial P_B}{\partial l}\Delta l \end{aligned} \qquad (4)^2$$

3. Estimation of y_r and k, and Factor Decomposition of Fixed Income and Equity Returns

3.1 Expected Equity Risk Premium

It is impossible to observe the expected risk premium of equity directly from the market. This article thus adopts an approach that indirectly estimates the expected equity risk premium from the fixed income risk premium. Specifically, the expected equity risk premium can be obtained using equation 5 by assuming that premiums per risk converges to the same value θ for both fixed income and equity, if the market is efficient. The volatilities of fixed income and equity returns were estimated based on historical data for the last 12 months. y_n was estimated by a current yield from NOMURA-BPI , while r_f was estimated by the overnight call rate (secured).

$$\theta = \frac{y_n - r_f}{\sigma_B}, k = \theta \cdot \sigma_S = \frac{\sigma_S}{\sigma_B}\left(y_n - r_f\right) \qquad (5)$$

There are various methods to estimate the expected risk premium of equity. For example, Suwabe (2003) estimated the expected risk premium implied by an actual cash value at each time using an equity price model (a residual profit model). In this case, the earnings forecast for all issues in TOPIX (the dividend forecast, in the case of a dividend discount model) is required for the time periods. This article does not employ earnings forecast data like that of Toyo Keizai because such data do not cover all issues in TOPIX[3].

However, the estimation of the expected risk premium seems to have coincidence with the result of Suwabe (2003). **Figure 1** shows the estimation, which stays at quite a low level during the late 1980s and early 1990s, known as the bubble years. Subsequently it returns to a steady level.

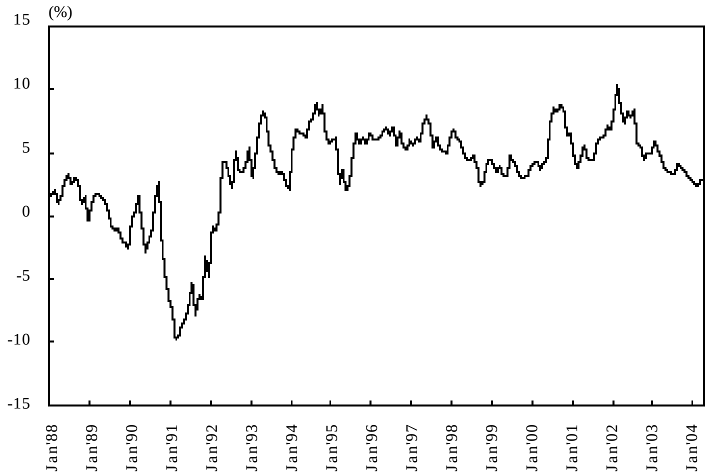
---

rate. So it was assumed that the effectiveness of a change in expected inflation rates could be ignored. Hence, the dividend for the next period is analyzed using ΔD_0, based on a nominal base (=real basis).

[2] *The return on income was expressed as $c + \partial P_B / \partial t = y_r + l$, when assuming a constant yield.*

[3] *Selecting an appropriate estimation method is difficult because the expected risk premium cannot be observed in the market. But section 5 shows that the expected risk premium is not regarded as the important factor in* the *estimation of correlation coefficients.*

Figure 1: Estimation of the Expected Equity Risk Premium



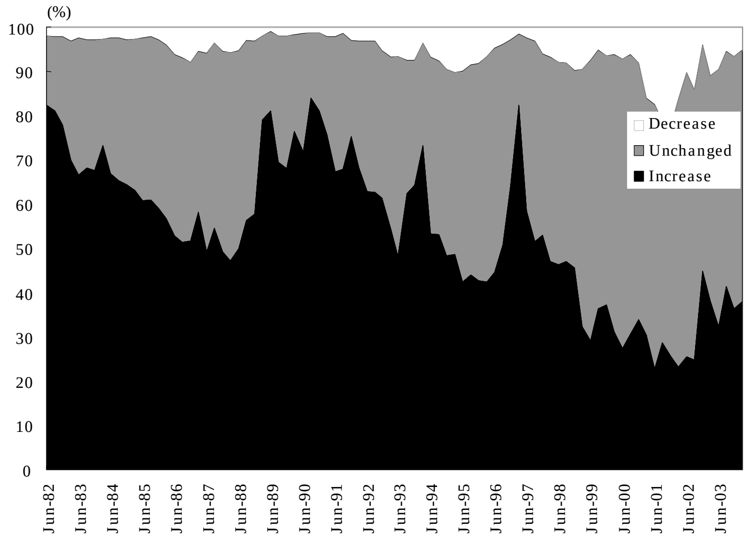
3.2 Expected Inflation Rates

Some studies in the United States try to estimate an expected inflation rate from the price of an inflation-linked bond. But in Japan, inflation-linked bonds have only been issued since March 2004, so such bonds are totally unavailable for analyzing the past long periods. Therefore, another approach is required and the following methods can be candidates: one using such time-series models as ARIMA or multivariate AR or one estimating the expected inflation rate from a survey of people about the expected inflation rate for the future.

As for the former method, a time-series model is difficult to construct, particularly in Japan where the prices of goods have been declining over 30 years. The ARIMA model was examined for this article. However it was found a negative expected inflation rate was unavoidable, so the model was unsuitable for our analysis. Kitagawa and Kawasaki (2001) have also tried to make a multivariate AR model, but they found it was difficult to fix the model by a popular statistical test like AIC, and selected explanatory variables were not stable as time progressed. Acknowledging that time-series models may be arbitrary, the following method, referred to as the Carlson-Parkin Method (CP method), was employed.

The CP method obtains the expectation and the standard deviation of a population distribution from the results of three questionnaires for the future conditions. As for a consumer price forecast, the Cabinet Office of Japan has been conducting a ‘Direction of Price’ survey (quarter-century data) included in the ‘Consumer Behavior Research’ questionnaire (see **Figure 2**). Quite a few analyses that employ the results of this questionnaire have been seen. Because such questionnaires about economic trend have some relationship with inflation expectations in the financial market, this article employs the results of these questionnaires.

Figure 2: 'Direction of Price' in 'Consumer Behavior Research'



Although the CP method has several variations, this article employs the adjusted CP method based on the rational expectations hypothesis proposed by Ogawa (1991), Doi (2001), and Hori and Terai (2004). The following is a brief description of the adjusted CP method. Assuming that respondent i holds a threshold δ_t when he/she forecasts a change in inflation rate, the expected inflation rate l_{it}^{r} formed by him/her is compared each time, not simply with the last realized inflation rate $l_{r,t}$, but with the threshold $l_{r,t}+\delta_t$. The respondent will answer, "increased" if the rate exceeds the threshold. Conversely, the respondent will answer, "decreased" if the rate falls below $l_{r,t}-\delta_t$. In addition, the respondent will answer, "unchanged" if the rate remains in $\left[l_{r,t}-\delta_t, l_{r,t}+\delta_t\right]$.

Assume here that the expected inflation rate l_{it}^{r} follows the normal distribution $N\left(\mu_t, \delta_t^2\right)$. Equation 6 can express the questionnaire results as follows: U_t as the percentage of respondents that answer the inflation rate would "increase", D_t as the percentage of respondents that answer "decrease". Φ means the distribution function for standard normal distribution.

$$
\begin{aligned}
U_t &= \Pr\left(l_{it}^{e} > l_{r,t}+\delta_t\right) = \Pr\left(\frac{l_{it}^{e}-\mu_t}{\sigma_t} > \frac{l_{r,t}+\delta_t-\mu_t}{\sigma_t}\right) = 1-\Phi\left(\frac{l_{r,t}+\delta_t-\mu_t}{\sigma_t}\right) \\
D_t &= \Pr\left(l_{it}^{e} < l_{r,t}-\delta_t\right) = \Pr\left(\frac{l_{it}^{e}-\mu_t}{\sigma_t} < \frac{l_{r,t}-\delta_t-\mu_t}{\sigma_t}\right) = \Phi\left(\frac{l_{r,t}-\delta_t-\mu_t}{\sigma_t}\right)
\end{aligned}
\tag{6}
$$

Defining u_t and d_t as follows. These values can be estimated from equation 6 when calculating

$\Phi^{-1}(1-U_t)$ and $\Phi^{-1}(D_t)$ respectively.

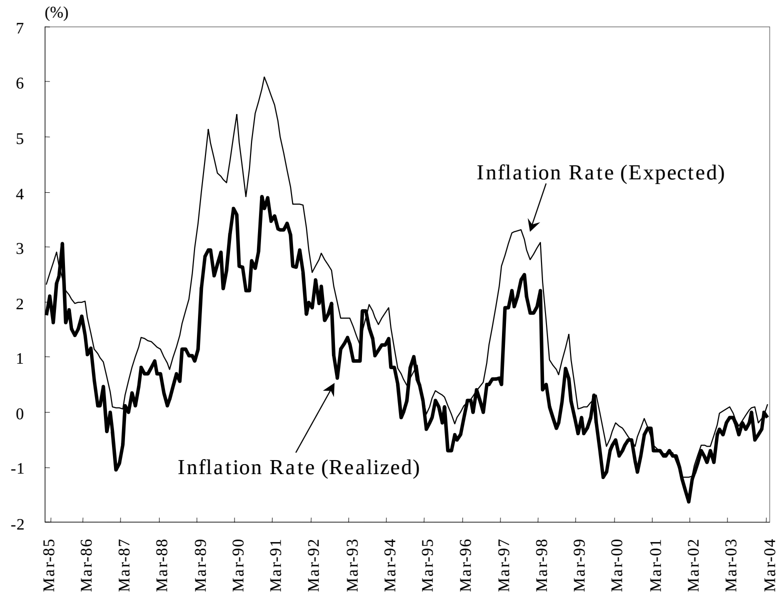
$$u_t \equiv \frac{l_{r,t} + \delta_t - \mu_t}{\sigma_t},\ d_t \equiv \frac{l_{r,t} - \delta_t - \mu_t}{\sigma_t} \quad (7)$$

If u_t and d_t are identified, μ_t and σ_t can finally be obtained from equation 8. Then μ_t is the estimation of expected inflation[4]. In addition, δ_t can be derived because the expected inflation l_{it}^e has the mean μ_t and the variance σ_t^2 [5].

$$\mu_t = l_{r,t} - \delta_t \frac{u_t + d_t}{u_t - d_t},\ \sigma_t = \frac{2\delta_t}{u_t - d_t} \quad (8)$$

Figure 3 shows the estimation of an expected inflation rate. The result is easy to understand when compared with the complicated derivation of the estimation formula. **Figure 2** and **Figure 3** implies that the expected inflation rate simply reflects the results of questionnaires. Although there is some fluctuation, those who answered, “increase from the current inflation rate” consistently account for the higher proportion of those surveyed.

Figure 3: Expected Inflation Rate by the Adjusted CP Method



[4] *Quarterly data is linearly complemented to convert into monthly data. Furthermore, it is believed that the expected inflation rate has a term structure; however, the article assumes that the rate is stable during the period.*

[5] *Refer to Ogawa (1991), Doi (2001), and Hori and Terai (2004) for details of the calculations.*

Figure 4: Nominal Interest Rate, Expected Inflation Rate, and Real Interest Rate

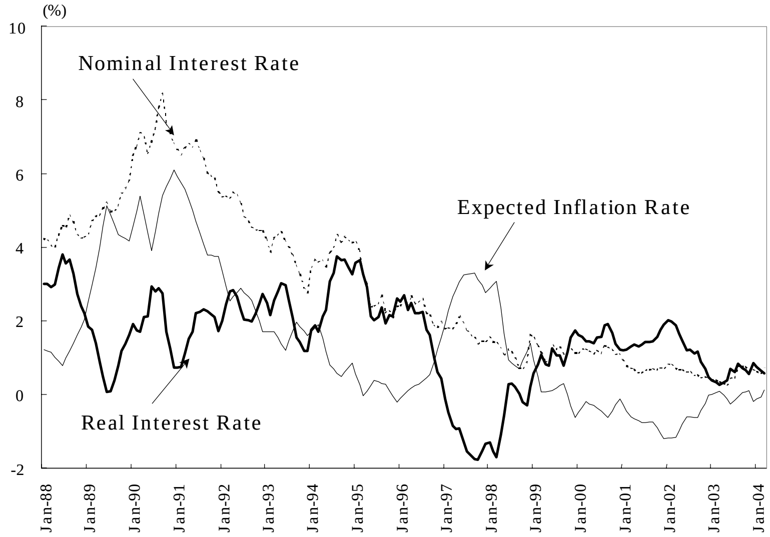


Figure 4 shows a real interest rate y_r obtained after deducting the expected inflation rate l by the adjusted CP method from the nominal interest rate y_n. The following points are noteworthy in the Figure: (1) most real interest rates fall within the 0% to 6% range and are relatively stable when compared with nominal interest rates, (2) real interest rates become negative between 1997 and 1998 because of the rises in expected inflations, and (3) not much difference has been observed between nominal and real interest rates since 2003.

3.3 Factor Decomposition Using Historical Data

By the method explained in sections 3.1 and 3.2, an expected equity risk premium and an expected inflation rate were estimated. Then, factor decomposition of fixed income and equity returns is conducted using the past data of equity and fixed income returns. These data represented approximately 16 years of monthly data from January 1988 to March 2004.

As for equity, **Figure 5** and **Table 1** show the factor decompositions for TOPIX returns r_s (with dividends). First, the following components showed a comparatively stable profit: real interest rate y_r, the part of income gains, and inflation rate l, the part of risk premium k and capital gains. Surprisingly k experienced an abnormal negative situation during 1990 to early 1992, but recovered and posted the highest accumulated return among all factors. On the other hand, real interest y_r has been broadly flat since 1997. Secondly, the capitals gains from y_r and k fluctuate more than the income gains from those factors. In particular, expected equity risk premium k fluctuated, which may affect an overall equity return significantly. Additionally, cumulatively, the capital gains from k and y_r realized positive profits.

However, the first and the second factors did not play an important role during the period. The most significant fluctuation is obviously shown by the expected dividend factor D (capital gain depends on changes in the dividend real growth rate g_r and expected dividends D_0 for the next period), the third factor. The expected dividend factor has consistently been on a downward trend since 1990. This is the only factor that has shown a negative return and the factor actually drives out all other factors. In other words, a downturn in an equity return is attributed to a reduction in the expected future dividends. This fact is consistent with the research results of Suwabe (2003) and others.

Figure 5: Factor Decomposition of Equity Return (Accumulated)

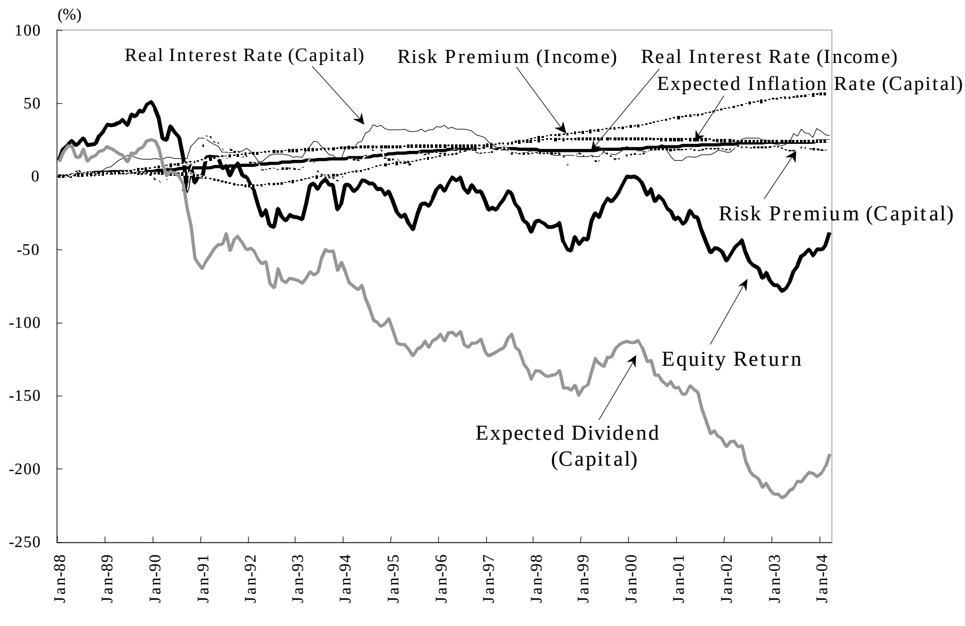


Table 1: Factor Decomposition of Equity Return (Annual Basis)

Equity	Real Interest Rate (Income)	Risk Premium (Income)	Expected Inflation (Capital)	Real Interest Rate (Capital)	Risk Premium (Capital)	Expected Dividend (Capital)
Mean	1.48%	3.49%	1.47%	1.72%	1.16%	-11.66%
Standard Deviation	0.35%	1.12%	0.54%	6.52%	9.79%	18.49%

As for fixed income, **Figure 6** and **Table 2** show the factor decomposition of NOMURA-BPI return. First, real interest rate y_r and inflation rate l, which are the same as those for equity as income gain, produce a stable profit. Secondly, the capital gains from the fluctuation of y_r and l are volatile, also similar as in the case for equity. Each has achieved return of around 10% cumulatively. This is because, different from equity, the price sensitivities to the fluctuations of y_r and l are always negative, and because y_r and l are continuously decreasing.

Figure 6: Factor Decomposition of Fixed Income Return (Accumulated)

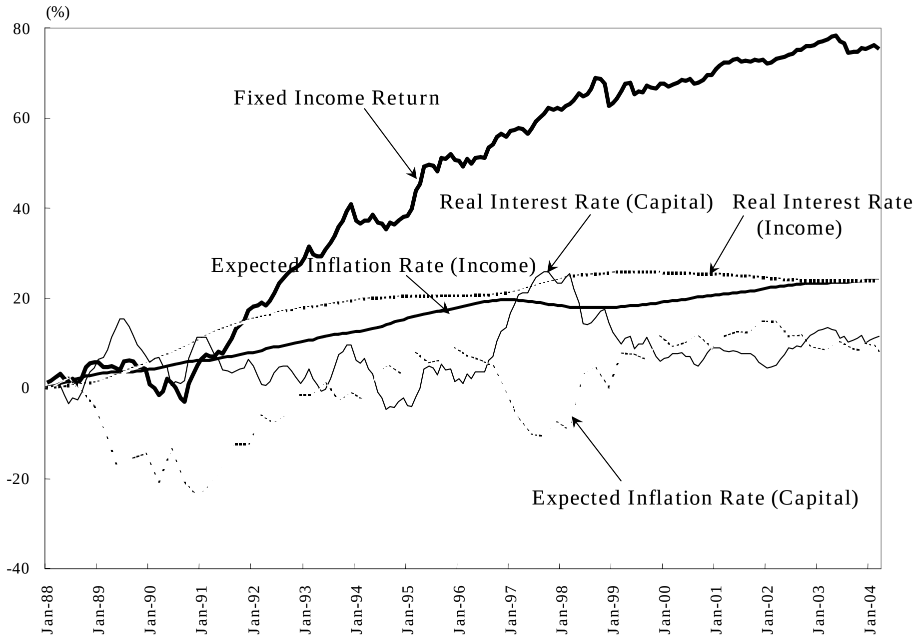


Table 2: Factor Decomposition of Fixed Income Return (Annual Basis)

Fixed Income	Real Interest Rate (Income)	Expected Inflation (Income)	Real Interest Rate (Capital)	Expected Inflation (Capital)
Mean	1.48%	1.47%	0.71%	0.51%
Standard Deviation	0.35%	0.54%	5.44%	4.48%

4. Factor Decomposition of Correlation Coefficients

In sections 2 and 3, fixed income and equity returns were decomposed into major factors. Here, the factor decomposition of a correlation coefficient between fixed income and equity returns is conducted. The correlation coefficient is given by equation 9.

$$\rho_{r_S,r_B} = \frac{\mathrm{cov}(r_S, r_B)}{\sigma_{r_S}\sigma_{r_B}} \tag{9}$$

After applying equations 3 and 4 to equation 9, let us assume that correlations between factors and their difference can be ignored. Then the numerator covariance can be decomposed into seven major factors (13 factors as minimum unit) as follows:

$$\begin{aligned}
&\mathrm{cov}(r_s, r_B) \\
&= \mathrm{cov}\left[y_r + k + l + \frac{\partial P_S}{\partial y_r}\Delta y_r + \frac{\partial P_S}{\partial k}\Delta k + D,\ y_r + l + \frac{\partial P_B}{\partial y_r}\Delta y_r + \frac{\partial P_B}{\partial l}\Delta l \right] \\
&= \left\{ \mathrm{var}(y_r) + \frac{\partial P_S}{\partial y_r}\frac{\partial P_B}{\partial y_r}\mathrm{var}(\Delta y_r) \right\} + \left\{ \mathrm{cov}(k, y_r) + \frac{\partial P_S}{\partial k}\frac{\partial P_B}{\partial y_r}\mathrm{cov}(\Delta k, \Delta y_r) \right\} + \left\{ \mathrm{cov}(D, y_r) + \frac{\partial P_B}{\partial y_r}\mathrm{cov}(D, \Delta y_r) \right\} \qquad (10) \\
&+ \mathrm{var}(l) + \left\{ 2\,\mathrm{cov}(y_r, l) + \frac{\partial P_S}{\partial y_r}\frac{\partial P_B}{\partial l}\mathrm{cov}(\Delta y_r, \Delta l) \right\} + \left\{ \mathrm{cov}(k, l) + \frac{\partial P_S}{\partial k}\frac{\partial P_B}{\partial l}\mathrm{cov}(\Delta k, \Delta l) \right\} + \left\{ \mathrm{cov}(D, l) + \frac{\partial P_B}{\partial l}\mathrm{cov}(D, \Delta l) \right\}
\end{aligned}$$

This equation makes it possible to decompose the correlation coefficient of equation 9 into seven factors. More precisely, (1) the real interest rate, (2) the expected equity risk premium and the real interest rate, (3) the expected dividend factor and the real interest rate, (4) the expected inflation rate, (5) the real interest rate and the expected inflation rate, (6) the expected equity risk premium and the expected inflation rate, and (7) the expected dividend factor and the expected inflation rate.

Before examining each factor, these factors will be grouped into two categories when we epitomizes them into the former three (1, 2, 3) and the latter four (4, 5, 6, 7) in order to observe an overall trend. The former group certainly has the variables related to the real interest rate y_r. So, the aggregated impact of the real interest rate on the correlation coefficient between fixed income and equity returns can be observed. Meanwhile, the latter group certainly has the variables related to expected inflation rate l, so the aggregated impact of the expected inflation rate on the correlation coefficient can be observed. The article refers to the former group as the broad real interest rate factor and to the latter group as the broad expected inflation factor.

5. Research Results[6]

Figure 7 and **Table 3** show the correlation coefficient (realized value), the correlation coefficient (estimation by seven factors), the broad real interest rate factor and the broad expected inflation factor. As Asano (2005) pointed out, the correlation coefficient between fixed income and equity returns, which had been positive and high, rapidly declined since 1991.

The estimated correlation coefficient by the factor decomposition method well describes the realized value, and shows partial divergence only in early 1990 and 1995. Both the broad real interest rate factor and the broad expected inflation factor show significant fluctuations and have periodicities. It is significant that they move in opposite directions. Additionally, the correlation coefficient, which moves between both factors, is observed.

Figure 7: Correlation Coefficient (Realized), Correlation Coefficient (Estimated), Broad Real Interest Rate Factor and Broad Expected Inflation Factor

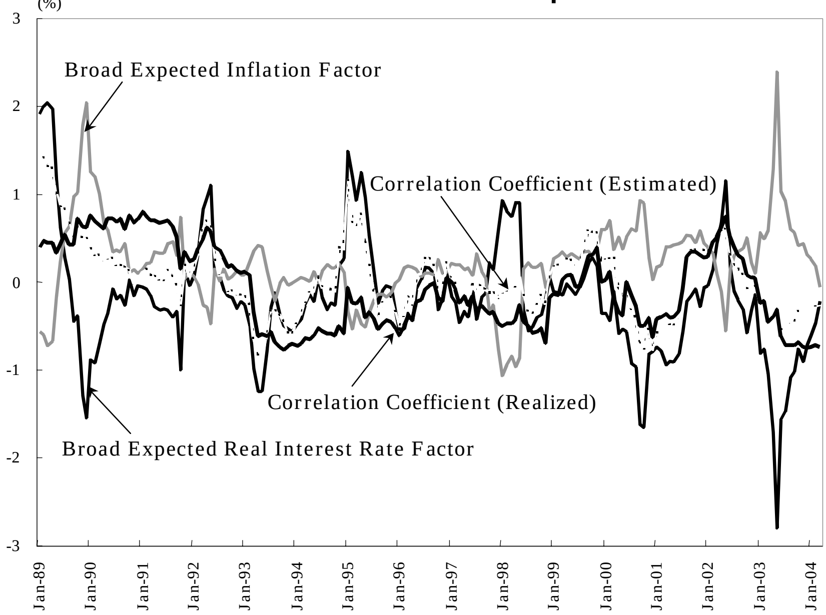


Table 3: Broad Real Interest Rate Factor and Broad Expected Inflation Factor

Factor	89/1	90/1	91/1	92/1	93/1	94/1	95/1	96/1	97/1	98/1	99/1	00/1	01/1	02/1	03/1
Real Interest Rate	2.006	-0.914	-0.069	0.333	-0.491	-0.422	1.214	-0.536	-0.219	0.813	-0.113	-0.441	-0.774	0.147	-0.756
Expected Inflation	-0.593	1.206	0.211	-0.022	0.238	0.051	-0.533	0.157	0.206	-0.936	0.298	0.703	0.196	0.329	0.492
Total	1.413	0.292	0.142	0.311	-0.253	-0.371	0.682	-0.379	-0.013	-0.123	0.185	0.263	-0.578	0.476	-0.264

[6] *Correlation coefficients, covariance, and variance are calculated using the 12 last months of data.*

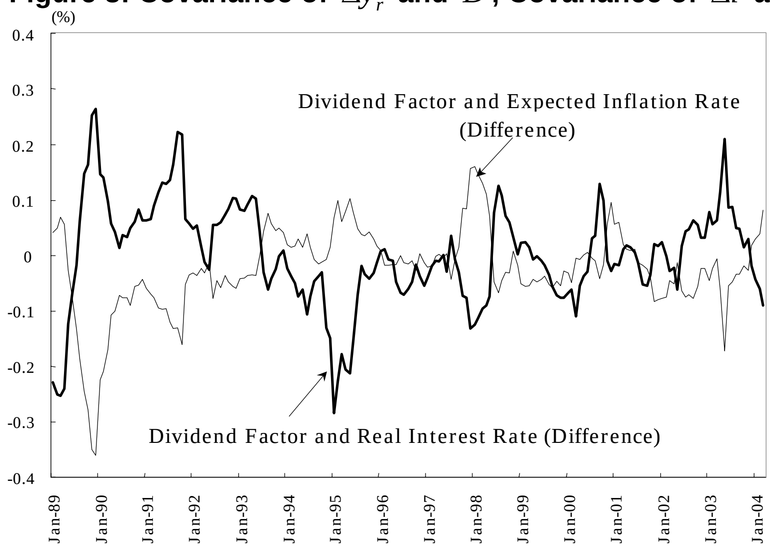
To analyze these factors more precisely, let us examine each of the seven factors. **Table 4** shows that the each factor has a different influence. However, factors Δy_r and D in equation 3, and factors Δl and D in equation 6 particularly have bigger influences.

Additionally, **Figure 8** shows the following covariances: the covariance of Δy_r and D having a significant influence in factor (3), the covariance of Δy_r and D having a significant influence in factor (6). Considering the relation of these covariances and two broad factors in **Figure 7** makes it possible to understand the specific interactions within an economy, which drives the correlation coefficient between fixed income and equity returns. For example, the correlation coefficient is boosted in a positive direction by an increase in the broad real interest rate factor, when the correlation between Δy_r and D declines just as during 1999 and late 2003. Conversely, when the correlation between Δl and D increases just as during 1998 and 2002, the correlation coefficient is reduced in a negative direction by a decrease in the broad real interest rate factor. Similarly, when the correlation between Δl and D declines (during 1998 and 2002), the correlation coefficient is boosted in a positive direction by an increase in the broad expected inflation factor. On the contrary, the correlation coefficient declines by a decrease in the broad expected inflation factor where the correlation between Δl and D increases.

Table 4: Seven Factors Constituting Correlation Coefficient

Factor	89/1	90/1	91/1	92/1	93/1	94/1	95/1	96/1	97/1	98/1	99/1	00/1	01/1	02/1	03/1
①	0.772	-0.200	0.263	0.631	-0.054	-0.636	-0.012	-0.477	-0.333	0.159	0.022	-0.778	-0.875	0.127	-0.428
②	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.003
③	1.233	-0.714	-0.332	-0.298	-0.437	0.214	1.222	-0.060	0.114	0.654	-0.135	0.337	0.100	0.019	-0.331
④	-0.352	0.163	-0.125	-0.240	0.031	0.140	0.006	0.061	0.111	-0.102	-0.017	0.418	0.528	-0.081	0.358
⑤	-0.001	-0.001	-0.001	-0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	-0.001	-0.002
⑥	-0.240	1.044	0.337	0.221	0.207	-0.089	-0.538	0.095	0.095	-0.834	0.315	0.285	-0.332	0.411	0.136
⑦	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.001

Figure 8: Covariance of Δy_r and D, Covariance of Δl and D



What do such movements in each factor imply? As equation 10 has shown, the correlation signs between Δy_r and D, and also between fixed income and equity returns are opposite because $\mathrm{cov}(D, \Delta y_r)$ is multiplied by $\partial P_B / \partial y_r$. For instance, let us consider the situation where y_r rises and D is improved. Thus the correlation between them increases. The improvement in D is directly linked to the increase in equity returns, whereas the increase in y_r has a negative effect on fixed income returns. As a result, the correlation between fixed income and equity returns decreases. Furthermore, equation 10 indicates that such processes can be observed similarly in Δl. Finally, the correlation between Δy_r and D becomes the determining factor for the correlation between fixed income and equity returns because the influence of the broad real interest rate factor exceeds that of the broad expected inflation factor[7]. In addition, the reason why correlations of Δy_r, D and Δl, D move in opposite directions as in **Figure 8** is the inverse correlation between y_r and l.

Incidentally, the factor related to the expected equity risk premium k hardly has any influence on a correlation coefficient. It can be explained by equation 10. As **Figure 5** and **Table 1** show, D has more significant fluctuation than other factors. Hence, the significantly fluctuating D and the variable that forms covariance with D have a great influence. That is the case for the real interest rate y_r and the expected inflation rate l, which have covariance with D. However, the expected equity risk premium k does not. That is why the expected risk premium k is not an important factor for the correlation coefficient. From those analyses, real interest rate Δy_r and D have been confirmed as the most important factors affecting the correlation coefficient between fixed income and equity returns.

To obtain real interest rate Δy_r, the nominal interest rate y_n or the expected inflation rate l is required. This article has employed the adjusted CP method to estimate expected inflation rates but another better approach might be required. Additionally, although the article has impliedly calculated the expected dividend D as an ex-post factor, other forecasting methods should be considered. However, the method employed in the article can be far simpler than merely accepting the calculated correlation coefficient from historical data, or than examining an enormous macro and micro factors one by one which become headlines of a newspaper.

6. Influence of Correlation Coefficients on ALM

Lastly, let's discusses the influence of correlation coefficients on assets and surplus in the pension ALM (Asset Liability Management). Given the fact that a real interest rate and an expected inflation rate are the most important factors affecting correlation coefficients as shown in section 5, this section also focuses on these two factors in conducting the analysis. This is because, fortunately, the interest rate and

[7] *Table 6 shows the sensitivity of the correlation coefficient (bonds and equity) for changes in the correlation coefficient (Δy_r and D).*

the expected inflation rate are also important factors for a pension liability returen. First of all, this section defines the return of pension liabilities based on Yano (2004). D_L (D_L =12) is the adjusted duration of pension liability, $y_r 12$ is the real interest rate on 12-year government bonds, $\phi(0 \geq \phi \geq 1)$ is the adjusting rate of benefit to inflation, and r_L is the return of a nominal liability. r_L can be expressed by the following equation:

$$r_L = y_r 12 + \phi l_r - D_L \{\Delta y_r 12 + (1-\phi)\Delta l\} \tag{11}$$

When equation 11 for a pension liability return is compared with equation 3 for equity return and equation 4 for fixed income return, the real interest rate is found to be the common factor among the three equations in terms of both income gain and capital gain. It implies that the correlations between two of them are high. Regardless of income gains and capital gains, the third term of equation 3, equation 4, and the second term of equation 11 show that the inflation rate is the common factor in the three equations with a positive contribution. It should also be noted that the inflation rate is realized in a pension liability return, while the rate is expected in fixed income return. Additionally, not the entire realized inflation rate, only some portion of it, is reflected in a pension liability return. On the contrary, the fourth terms in equation 11 for a pension liability and in equation 4 for fixed income show that an increase in the expected inflation negatively affects returns. However, the more a pension benefit adjust to inflation, the less a pension liability return fluctuates.

Figure 9 and **Table 5** show factor decomposition for the cumulative return of a pension liability[8]. The real interest rate y_r12 (income gains) and the realized inflation rate l_r generate stable profits. In total, they yield around 30% and 6% cumulative returns respectively. Additionally, the capital gains caused by the real interest rate y_r12 and the realized inflation rate l_r fluctuate significantly when compared with income gains, just as in the cases of fixed income and equity. Each of the terms eventually holds a positive cumulative return. However, the profit caused by the expected inflation rate l is relatively low compared with that by the real interest rate y_r12 during the period. The real interest rate y_r12 yielded approximately 30% return, while the expected inflation rate l over 6% return.

Figure 9: Factor Decomposition of Pension Liability Return (Accumulated)

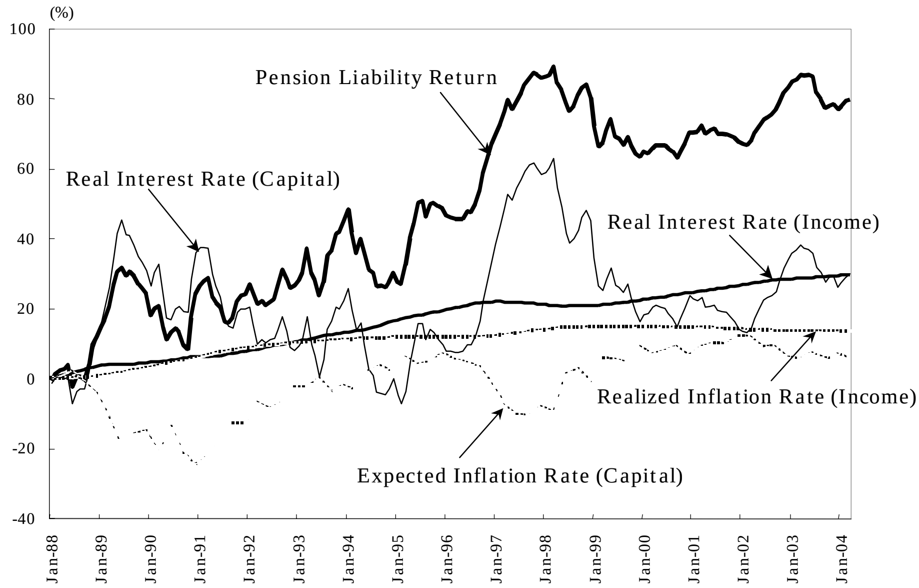


Table 5: Factor Decomposition of Return of Pension Liabilities (Annual Basis)

Pension Liabilities	Real Interest Rate (Income)	Realized Inflation (Income)	Real Interest Rate (Capital)	Expected Inflation (Capital)
Mean	1.85%	0.38%	1.85%	0.42%
Standard Deviation	0.37%	0.20%	12.72%	5.05%

[8] *Figure 9 shows the case of 0.5% in adjusting the rate of inflation. The national bond yield applied in Equation 11 was obtained from Factset. 10-year data was used as an alternative because 12-year data was unavailable.*

The surplus formula can be given by assets and a pension liability. When the difference between assets (S as equity, B as fixed income) and pension liability L is surplus Y, and a ratio (funding ratio) for the pension liability of assets is f, the formula is expressed as follows:

$$Y = (S + B) - L,\ f = \frac{S + B}{L}$$

When surplus return ΔY is expressed as a rate of return (standardizing it with pension liability L) and w represents the equity weight in assets, surplus return r_Y can be defined as follows:

$$r_Y = \{w r_S + (1 - w) r_B\} f - r_L \qquad (12)$$

In addition, surplus risk σ_Y can be obtained by the following formula:

$$\begin{aligned} \sigma_Y^2 &= w^2 f^2 \sigma_S^2 + (1-w)^2 f^2 \sigma_B^2 + \sigma_L^2 \\ &+ 2w(1-w) f^2 \operatorname{cov}(r_S, r_B) - 2(1-w) f \operatorname{cov}(r_B, r_L) - 2wf \operatorname{cov}(r_S, r_L) \\ &= \{w^2 f^2 \sigma_S^2 + (1-w)^2 f^2 \sigma_B^2 + 2w(1-w) f^2 \operatorname{cov}(r_S, r_B)\} \\ &+ \{\sigma_L^2 - 2(1-w) f \operatorname{cov}(r_B, r_L) - 2wf \operatorname{cov}(r_S, r_L)\} \end{aligned} \qquad (13)$$

Dopfel (2003) has mentioned that a decreasing correlation affects the entire risk differently in the case of assets and the case of surplus including assets and debts. Such difference is easily understood, because the first term in equation 13 is the risk of assets and the second term added to the first term means the surplus risk[9]. The decrease in the correlation between fixed income and equity returns causes a reduction in risk of the third factor of the first term. On the contrary, the correlation decrease in the third factor of the second term may increase risk because equity is more likely to have a different feature to a pension liability, which is similar to fixed income.

A surplus simulation was conducted by changing correlation coefficients or the return and the risk of a surplus. Parameters in the standard model are set as follows: 0.5 $(\phi = 0.5)$ as an adjusting rate to inflation, 1 ($f = 1$) as the funding ratio, and 60% for equity weight and 40% for fixed income weight $(w = 0.6)$. Parameters were randomly changed from these first settings.

[9] *The first term is referred to as the risk among assets and the second term as the risk between assets and pension liabilities.*

Table 6: Three Different Correlation Coefficients and Sensitivity to Change in Correlation Coefficient Between Δy_r and D

Adjusting Rate	Sensitivity for Δ Corr ($\Delta y_r, D$)			Mean		
	Δ Corr (r_S, r_B)	Δ Corr (r_B, r_L)	Δ Corr (r_S, r_L)	Corr (r_S, r_B)	Corr (r_B, r_L)	Corr (r_S, r_L)
0	-0.3129**	-0.1291*	-0.1499**	-0.0596	0.4185**	0.0342
0.5	-0.3129**	-0.1346*	-0.3822**	-0.0596	0.3204**	-0.0533
1	-0.3129**	-0.1121	-0.4786**	-0.0596	0.2094**	-0.1070

** significant at the 2.5% level; *significant at the 5% level.

The left part of **Table 6** shows the sensitivities of three correlation coefficients for a change in the correlation coefficient between Δy and D, when adjusting rates of inflation were 0, 0.5, and 1. First, it was found that the sensitivity of the correlation coefficient (r_S and r_B) for a change in the correlation coefficient between Δy and D was negative. This result supports section 5, which shows that the correlation between equity and fixed income returns decreases when the correlation between Δy_r and D increases. In addition, because the correlation coefficient is between assets, it is clearly independent of the inflation-adjusting rate. Secondly, a strong relationship was not identified in the correlation coefficient (r_B and r_L) for a change in the correlation coefficient between Δy and D. Thus, the correlation between r_B and r_L is not much influenced by the movement in the correlation between y_r and D. Thirdly, it was found that the sensitivity of the correlation between r_S and r_L to a change in the correlation between Δy_r and D, was negative. Thus, the correlation coefficient between equity and pension liabilities returns decreases as the correlation coefficient between equity and fixed income returns decreases. This result showed that the hedge efficiency of equity that has some similarity with fixed income, decreased for a pension liabilitiy, as Dopfel (2003) has already pointed out.

Interestingly, when inflation rate ϕ increased, it was observed that the negative tendency of the correlation between equity and pension liability returns was strengthened, when the correlation between Δy_r and D increases. This phenomenon can similarly be described in section 5 where the correlation between fixed income and equity increases along with increases in y_r and D. Because the improvement in D directly increases equity returns, the remaining relationship between Δy_r and r_L needs to be considered. As shown in equation 11, where the adjusting rate of inflation ϕ is 0, a capital gain is significantly influenced not only by Δy_r but also by Δl. However when ϕ is 1, the fluctuation is only dictated by y_r, thus negative relationships clearly appear in r_L.

Table 7: Returns and Risks of Surpluses in Pension ALM (Constant Real Interest Rate Case)

Parameter	Value	Return (Mean)	Risk (Standard Deviation)		
			Asset	Liability	Surplus
Inflation Adjusting Rate	0	-0.0589	0.1151	0.0641	0.1391
	0.5	-0.0616	0.1151	0.0868	0.1485
	1	-0.0643	0.1151	0.1188	0.1689
Funding Ratio	0.5	-0.0564	0.0575	0.0897	0.1092
	1	-0.0616	0.1151	0.0868	0.1485
	1.5	-0.0669	0.1726	0.0829	0.1965
Equity Weight	0.8	-0.0812	0.1528	0.0881	0.1814
	0.6	-0.0616	0.1151	0.0868	0.1485
	0.4	-0.0421	0.0788	0.0850	0.1195

Assuming that a real interest rate remains constant, let us consider the change in the surplus risk with variously changed parameters of a pension liability and assets (**Table 7**). First, the surplus risk increases when the adjusting rate of inflation ϕ rises. Along with the increase in the adjusting rate, the correlation between r_B and r_L decreases keeping a positive value, while the correlation between r_S and r_L decreases turning to a negative value[10]. Hence, the risk between assets and a pension liability increases because the second value of the second term in Formula 13 decreases the negative degree, while the third value in the term also changes from negative to positive.

Secondly, different from the adjusting rate of inflation, when the funding ratio changes, it causes no direct change in a pension liability return. Thus, the ratio does not have any effect on each correlation coefficient. **Table 7** shows that the risk between assets increases with increase in the funding ratio, because the first term depends entirely on the funding ratio. However no significant change is identified in the risk between assets and a pension liability. This is because the influences of the funding ratio on the second and third values in the second term are cancelled out because these value signs are opposite under the conditions of the standard model[11].

Thirdly, as the funding ratio, the equity weight does not have a direct effect on a correlation coefficient. When the weight of equity with high risk decreased, the risk between assets decreased in the aggregate. As for the risk between assets and a pension liability under standard conditions, the third value in the second term becomes positive, but this portion reduces with the decrease in equity weight. Additionally, because the second value of the second term becomes negative, and because the weight in this portion increases in contrast with the reduction in equity weight, the value decreases further. Finally, the risk between assets and pension liabilities decreases.

[10] *See the first and second column from the right in Table 6.*
[11] *See the middle line of the first and second column from the right in Table 6.*

However, it should be noted that where the correlation between Δy_r and D changes, such a discussion based on the historical data is not always conclusive. As explained earlier, this is because the values or signs of three different correlation coefficients (the right side of **Table 6**) easily vary because of the change in the correlation between Δy_r and D. In the surplus risk, the fluctuation of correlation coefficients between fixed income and equity returns has the potential for considerable impact not only on the risk between assets but also on the risk between assets and liabilities.

7. Conclusion

This article employed actual market data to conduct factor decomposition of equity and fixed income returns. Then the article decomposed the correlation coefficient between fixed income and equity returns into several factors and examined each. As a conclusion, the article confirmed that the correlation between real interest rates and expected dividends has the most significant influence. Additionally, as for a pension liability return, the article employed the surplus relationship, where a real interest rate and an expected inflation rate are important factors, to examine the following points: how the correlation coefficient among fixed income, equity, and a pension liability varies with the change in the correlation between real interest rates and expected dividends and what makes a correlation coefficient which influences surplus risk change.

The method attempted in the article is the factor analysis of returns, but this method involves some issues including the interpretation of the research result by historical data. One is the estimation method of expected inflation rates. The article employed the adjusted CP method, because such an asset as inflation-linked bonds, which allows us to know expected inflation rates directly, is not available. However, since there is no approach to confirm the true expected inflation rate for now, more research is required. The other one is the state of the economy. According to Ilmanen (2003) and Asano (2005), the state of the economy can be recognized as an important factor affecting the correlation coefficient between fixed income and equity returns. The article used long-term data for over 16 years for the analysis. During these years, except for a few years at the beginning, nominal interest rates constantly declined and the Japanese economy remained sluggish. Hence, it is undeniable that a different result may be obtained if there is significant change in the economy in the future. Further research and analysis are required, including completely different approaches from what was adopted in the article.

I would like to express my gratitude to Yukihiro Asano (Yokohama National University) who gave many useful comments. The author is solely responsible for the content of this paper, and any opinions expressed and errors are his alone.

References:

Arnott, Robert D. and Petrer L. Bernstein, "What Risk Premium Is 'Normal'?" *Financial Analysts Journal*, March/April 2002.

Asano, Yukihiro, "The Correlation between Equity and Fixed Income," *Sumitomo Pension Journal*, Vol. 5 May 2005 (in Japanese).

Campbell, John Y. and Tuomo Vuolteenaho, "Inflation Illusion and Stock Prices," *NBER Working Papers* 10263, National Bureau of Economic Research.

Doi, Takero, "Increase in Saving Rate and Precautionary Saving Motive in Japan," *ESRI Discussion Paper Series* No.1, 2001 (in Japanese).

Dopfel, Frederick E., "Asset Allocation in a Lower Stock-Bond Correlation Environment," *The Journal of Portfolio Management*, Fall 2003.

Hori, Masahiro and Noboru Terai, "Applications of the Carlson-Parkin Method to Japanese Price Expectations," *ESRI Discussion Paper Series* No. 91, 2004 (in Japanese).

Ibbotson, Roger G. and Peng Chen, "Long-Run Stock Returns: Participating in the Real Economy," *Financial Analysts Journal*, January/February 2003.

Ilmanen, Antti, "Stock-Bond Correlations," *The Journal of Fixed Income*, September 2003.

Kitagawa, Genshiro, and Yoshinori Kawasaki, "Analysis in Prediction Error of Inflation by Time Series Model," *Bank of Japan Working Paper* 01-13, 2001 (in Japanese).

Miyoshi, Yasuo, "Long-Term Equity Return Estimation Considering Changes in Valuation," *Security Analysts Journal* Vol.41, No.5, May 2003 (in Japanese).

Ogawa, Kazuo, "Income Risk and Precautionary Saving," *The Economic Review* Vol. 42, No.2, April 1991 (in Japanese).

Sangyong Joo, "Stock Returns and Inflation: A Covariance Decomposition," *Applied Economics Letters*, April 2000, Vol. 7, Issue 4.

Suwabe, Takashi, "Global Convergence of Equity Risk Premium," *Ageing and Pension Investment* Vol. 22 No. 4, December 2003 (in Japanese).

Yano, Manabu, "Pension ALM with Inflation-Protected Bonds," *Security Analysts Journal* Vol.42, No7, July 2004 (in Japanese).

Yamaguchi, Katsunari, Yoshio Kanasaki, Akio Makabe, and Tadaaki Komatsubara, "Equity Risk Premium in Japan – Empirical Evidence over 50 years from a Supply-side Perspective", Paper presented at 11th Annual Conference of Nippon Finance Association, June 2003(in Japanese).