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Premium for Upside Risk and Downside Risk: decomposition of volatility effect

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Abstract

This paper studies the characteristics of upmarkets (positive return) and downmarkets (negative return) based on the trade-off relationship of risk and return. In seeking a reference point (based on which upmarkets and downmarkets can be determined) among the mean, zero, risk-free, and benchmark (relative) return, we find that zero, which is consistent with prospect theory, is the most appropriate. We confirm that in downmarkets, risk/return trade-off exists, but it becomes opposite in upmarkets—the volatility effect could be a compound effect of the two phenomena.

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Contents

1. Introduction

The volatility effect in equity markets, where stocks with higher volatility earn lower returns, has been well documented: Blitz and Vliet [2007], Yamada and Uesaki [2009], Ishibe et al. [2009]. These studies suggest that, when one considers volatility as a risk, the theoretical trade-off relationship between risk and return is not established in practice, which is very interesting. Moreover, this effect is considered to be one of the reasons why a minimum variance portfolio, like a minimum volatility portfolio, is more efficient for cap-weighted indexes (TOPIX etc.)（Ishibe et al.[2009]）.

Then, what type of risk should we expect a premium for? One of the candidates is downside risk. Ang et al. [2002] documents a trade-off relationship between downside correlations and forthcoming returns. Ang et al. [2006] treats downside beta. Amenc et al. [2010] discusses semi-deviation (SEM) as a downside risk measure for efficient indexation. This theme is also an interesting subject in behavioral economics, because SEM is a risk concept that is most akin to loss aversion under prospect theory (Kahneman and Tversky [1979]).

Is a risk volatility, or loss? Do investors behave differently on the upside of a market from the downside? Based on such questions, we examine the relationship between downside risk, upside risk, volatility (standard deviation), and equity risk premium. In addition, our analyses apply the prospect theory-like assumption where there are bounded rational investors, not a CAPM-like assumption where all investors act rationally.

2. Data and Methodology

There are many kinds of downside risk. Being based on a lot of distributional information, well known VaR and CVaR would be too complicated for bounded rational investors, and hence we adopted more intuitive SEM as a downside risk measure.

For a negative return's mean-like measure of downside risk, we define SEM in equation (1). The expectation assumes it is a sample mean. If total stock return in period t minus the target return is smaller than zero, we replaced it with zero, not excluded it. Furthermore, we define upside risk which replaced the minimum in equation (1) with a maximum. This upside risk is equivalent to a positive return's mean-like measure.

$$SEM_i = \sqrt{E\left[\left\{\min(r_{i,t} - \tau, 0)\right\}^2\right]} \qquad (1)$$

$r_{i,t}$: i - th total stock return in period t

τ: the target return

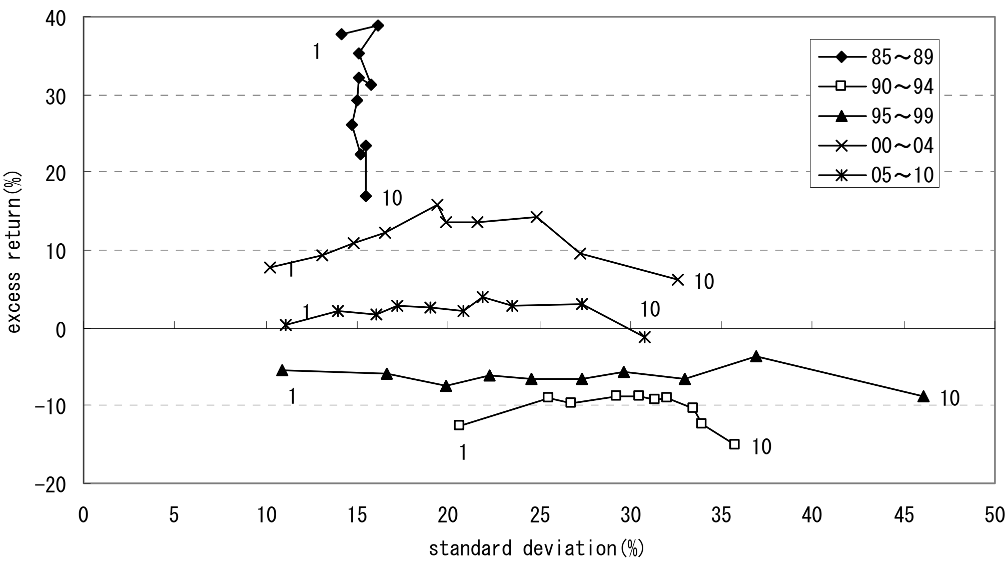
The target return τ, defining lower and upper bound, is breakeven point between gain and loss, and is regarded as a reference point in prospect theory because of having the same meaning. (The SEM function is defined on deviations from a reference point.)

The data set employed in this paper consists of monthly returns of stocks listed on the Tokyo Stock Exchange (TSE) 1st Section from January 1980 to December 2010 in the Nikkei NEEDS database. Volatility, downside risk, and upside risk are calculated based on data over the last five years' (past 60 months) monthly returns. Realized returns are annualized excess returns over risk-free returns, which are the uncollateralized overnight call rate (monthly mean) from *Tanshi-Kyokai*.

Exhibit 1 illustrates the relationship between realized standard deviations and excess returns. At the end of each month from December 1984 to November 2010 we constructed equally-weighted decile portfolios by ranking stocks based on the risk measures. Each decile portfolio included the same number of stocks. Stocks with the lowest ex-ante volatility (standard deviation) were assigned to the first decile, while those with the highest ex-ante volatility the last. We recorded excess return for each following one month. Each plotted line shows the standard deviation and mean of these realized excess returns for five-year periods (after 2005, six years); each line connects plot points from the low decile to high decile for the period concerned. From January 1985 to December 1989, the relationship between risk and excess return is not clear. Except for this period, one characteristic (hereafter 'continuity of risk') was observed for all periods, namely sequential consistency of realized standard deviation and ex-ante volatility. We omit the figures, but downside risk and upside risk were also similar.

In this paper, we analyze these relationships using the standard ranking portfolio method and the intersectional ranking portfolio method. The former divides the above-mentioned decile portfolios by volatility and SEM, and the latter analyzes average monthly returns on portfolios based on two risk measure quartiles. Therefore, the intersectional ranking portfolio method treats 16 portfolios at the same time.

Exhibit 1 Volatility Rank Analysis (sub-periods)



3. Results

Target return of risk characterization (τ)

First, we consider the target return τ of equation (1).

We examined 4 candidates for τ with the decile rank portfolio analysis. Average return of an individual stock（hereafter 'mean'）used in previous studies, Zero return（hereafter 'zero'）, Risk-free return（hereafter 'risk free'）, and Benchmark（a market index, like TOPIX）return（hereafter 'relative'）. We used Dividend included TOPIX for a benchmark.

Exhibit 2 examines the adequacy of each target return except the risk-free one. Because zero and risk free could not be distinguished on a graph, they were represented in zero. The all row shows mean subsequent returns for equal-weighted downside risk decile portfolios. For zero and relative, trade-off is established and regression coefficients are large (mean 0.05, zero 0.25, risk free 0.25, relative 0.20). Mean resembles a result of volatility (Exhibit 1), but trade-off is not established. In addition, since the difference in returns and standard deviation at zero is large, we think the zero standard can best distinguish the difference. Therefore, we use zero for the target return (τ).

Zero can expect a good property from the viewpoint of recognition

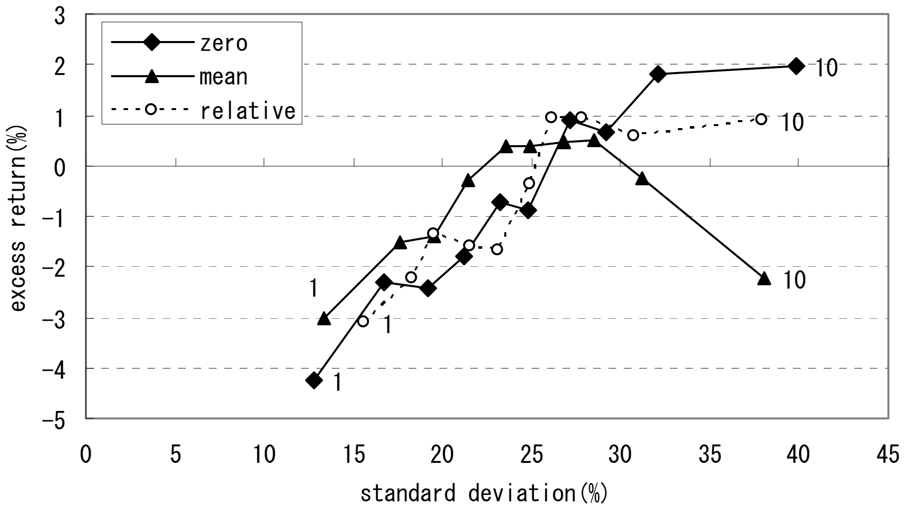
In zero target return, we assume negative return a downside risk and positive return an upside risk, which is easy to understand intuitively, and probably accessibility (Kahneman [2002]) is high[1]. Furthermore, among four candidates, a zero standard is the most consistent with loss aversion under prospect theory.

In contrast, with mean target return, it depends on investors what kind of average one takes and accessibility is low because one needs cognitive processing to calculate the average.

There are a lot of articles on mean target analysis in previous studies of downside risk. Because mean target downside risk (the mountain-shaped line in Exhibit 2) resembles volatility risk (Exhibit 1), we think any argument that tries to distinguish volatility risk from downside risk is difficult. As for relative target return, there are problems because plural candidate benchmarks exist, investors interested in a benchmark index are limited to institutional investors, and general recognition of a benchmark is low. We could substitute with zero for recent Japan, but zero is dominant because of ease of intuitive and cognitive processing.

[1] The ease with which particular mental content comes to mind. For example, similarity is more accessible than probability, changes are more accessible than absolute values, and averages are more accessible than sums.

Exhibit 2　Downside Risk Ranking Based on Three Target Returns (Whole　period)



Result of decile analyses using three risk criteria

We examined the situation according to economic environment. Exhibit 3 provides decile rank portfolio analysis sorted by three risk criteria (downside risk, upside risk, and volatility) for five-year periods.

According to downside risk, a trade-off is established because a return difference with the 10th rank and 1st rank and regression coefficient are positive in all periods. During 1990 through 1994 and 2000 through 2004, which covers the collapse of the bubble economy, return difference is large and the regression coefficient significant[2]. On the other hand, with upside risk and volatility, a trade-off is not established and the 10^{th} rank was the lowest return in all periods. In Exhibit 4, we can clearly confirm for all periods that the above-mentioned relationship and volatility are located in the middle of downside risk and upside risk.

Continuity of risk is established for all periods, except a reversal in the 5th and 6th under upside risk from 1990 through 1994.

[2] We think this result is consistent with the reversing behavior of risk aversion in the case of gains or risk seeking in the case of losses, so it has some relation to the reflection effect of prospect theory （and asymmetry between profits and losses）.

Exhibit 3 Rank Portfolio Analysis Using Three Risk Criteria (sub-periods)

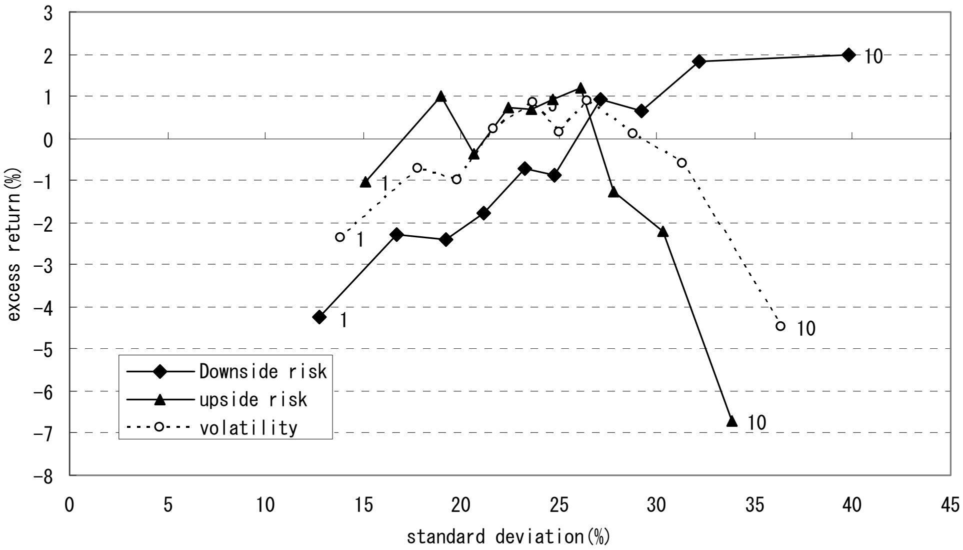
(A)90 ~94	Downside risk		upside risk		volatility	
	return	standard deviation	return	standard deviation	return	standard deviation
1	−16.90	17.84	−9.65	22.91	−12.70	20.61
2	−12.66	23.62	−7.46	26.88	−9.13	25.50
3	−13.75	26.36	−7.04	28.08	−9.74	26.81
4	−10.88	28.18	−9.32	29.14	−8.78	29.21
5	−10.15	30.15	−9.32	29.91	−8.88	30.48
6	−10.47	31.49	−9.82	29.59	−9.17	31.34
7	−7.79	33.49	−8.91	32.12	−9.00	32.02
8	−8.86	34.49	−12.21	33.02	−10.48	33.49
9	−7.06	35.30	−13.80	33.08	−12.30	33.95
10	−6.68	38.95	−17.66	34.43	−15.03	35.78
10−1	10.22	21.11	−8.01	11.52	−2.33	15.18
regression coefficient (t value)	0.50	(10.37)	−0.63	(−2.58)	−0.09	(−0.56)

(C)00 ~04	Downside risk		upside risk		volatility	
	return	standard	return	standard	return	standard
1	4.07	9.74	8.81	10.69	7.67	10.23
2	6.44	12.21	12.36	13.74	9.42	13.14
3	10.29	14.77	11.06	15.49	10.94	14.77
4	9.35	16.03	14.53	17.71	12.16	16.55
5	12.10	18.20	15.64	19.02	15.75	19.45
6	12.71	19.26	14.23	20.56	13.69	19.84
7	14.65	21.58	14.75	21.89	13.59	21.64
8	14.35	25.30	10.37	23.97	14.20	24.86
9	15.77	28.64	7.72	25.77	9.59	27.20
10	13.54	34.86	3.82	31.21	6.26	32.60
10−1	9.47	25.13	−4.98	20.52	−1.41	22.37
regression coefficient (t value)	0.40	(4.04)	−0.28	(−1.46)	−0.03	(−0.20)

(B)95 ~99	Downside risk		upside risk		volatility	
	return	standard deviation	return	standard deviation	return	standard deviation
1	−3.94	10.56	−5.65	12.46	−5.41	10.87
2	−3.77	14.79	−5.61	18.75	−5.83	16.66
3	−7.50	18.20	−7.91	20.83	−7.47	19.92
4	−7.32	21.72	−5.43	23.95	−6.03	22.32
5	−7.14	24.62	−7.19	25.21	−6.54	24.55
6	−8.05	26.88	−4.51	27.62	−6.46	27.32
7	−7.94	30.43	−5.86	28.04	−5.70	29.60
8	−6.61	33.20	−4.97	30.56	−6.60	33.01
9	−8.39	37.85	−5.00	36.20	−3.57	36.88
10	−1.64	51.82	−10.17	40.93	−8.70	46.08
10−1	2.30	41.26	−4.53	28.47	−3.30	35.21
regression coefficient (t value)	0.03	(0.39)	−0.07	(−0.97)	−0.03	(−0.72)

(D)05 ~10	Downside risk		upside risk		volatility	
	return	standard	return	standard	return	standard
1	−0.85	11.22	1.85	11.43	0.45	11.14
2	0.28	14.44	4.11	14.37	2.08	13.98
3	0.70	15.86	1.95	16.50	1.76	16.03
4	1.11	17.32	2.73	17.56	2.91	17.18
5	1.85	18.71	3.19	18.99	2.72	19.03
6	1.73	20.25	3.37	20.58	2.16	20.81
7	4.09	22.17	4.27	21.80	4.01	21.85
8	3.26	23.55	1.21	23.50	2.79	23.47
9	6.12	26.77	1.55	26.23	3.13	27.34
10	2.55	32.84	−3.42	29.14	−1.16	30.76
10−1	3.41	21.61	−5.27	17.71	−1.61	19.62
regression coefficient (t value)	0.24	(3.18)	−0.24	(−2.09)	−0.03	(−0.29)

Exhibit 4 Rank Portfolio Analysis Using Three Risk Criteria (Whole period)



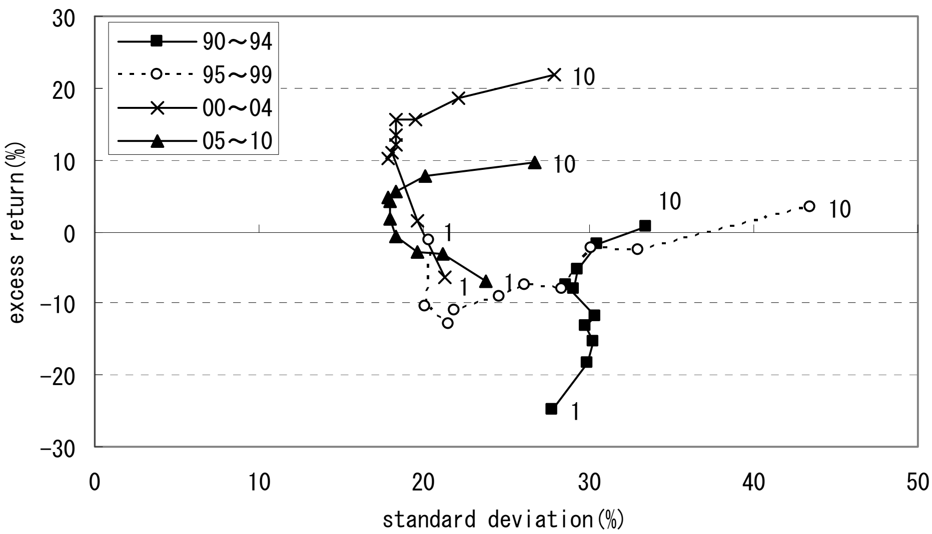
Relationship with return reversal

Because downside risk is the mean of past negative returns, the fact that realized return is big if downside risk is big suggests that there is a close relationship with return reversal where realized return is high when past mean return is low. Therefore, we conducted decile ranking portfolio analysis sorted by the mean of past 60 months' returns—the 1st rank for maximum past return, the 10th rank for minimum past return.

Exhibit 5 provides ranking portfolio analysis according to rank for the mean of past returns. Except for the 1st rank from 1995 through 1999, realized return is high so that rank is high, and the tendency is more marked than downside risk.

This result is consistent with an interpretation of mean return ≒ upside risk (a mean of positive return) − downside risk (a mean of negative return). A low past return means the combination of low upside risks and high downside risks, and the realization return is high for both, similarly a high past return means the combination of high upside risks and low downside risks, and the realization return is low for both. We cannot confirm continuity of risk according to the ranking for the mean of past returns. After 2000, standard deviation is large for ranking at both ends (like 1st, 10th) and it traces a bow shape. There are many stocks with larger upside risk for the 1st ranking and there are many stocks with larger downside risk for the 10th, therefore we can say that standard deviation becomes higher for both. In other words, we can explain return reversal as a compound effect of downside risk and upside risk.

Exhibit 5 Rank Portfolio Analysis Using Mean of Past Returns



Risk purification analysis

Because downside risk ignores upside risk in the calculation process, there may be stocks with high upside risk and high downside risk, and vice versa. We thus tried to remove stocks that persistently exhibited high downside risk and also those with substantial upside risk.

We pick stocks that consecutively appear in the same ranking. For example, for 1st ranked stocks, we selected those featuring as 1st rank for downside risk and also 1st rank for upside risk. Then, we classified into three: downside risk stocks, upside risk stocks, and overlapping stocks. For other ranked stocks, we classified similarly. The mean number of overlapping stocks was 86 for the 1st ranking, 73 for the 10th ranking, and 30 for the mean of the 2nd through 9th rankings (the mean of each stock ranking was 135). Because there are many overlapping stocks in the top and last ranks, this trial of risk purification is particularly effective for the 1st and 10th ranks.

The results are given in Exhibit 6. Compared to Exhibit 4 before purification, returns of the 10th ranking increased greatly in downside risk ranked portfolios—reverse trade-off characteristics where high risk stocks mean low return became clearer with upside risk ranked portfolios. However, no clear tendency is seen with respect to overlapping stocks.

Exhibit 7 is a summary of rank portfolio analysis. Based on discussion so far, we can confirm, in addition, that when upside risk purifies it, the range of standard deviation becomes half (risk differences in Exhibit 7). This perhaps suggests that upside risk does not influence realized standard deviation like other risks.

Exhibit 6　Rank Portfolio Analysis after Overlap Adjustment (Whole period)

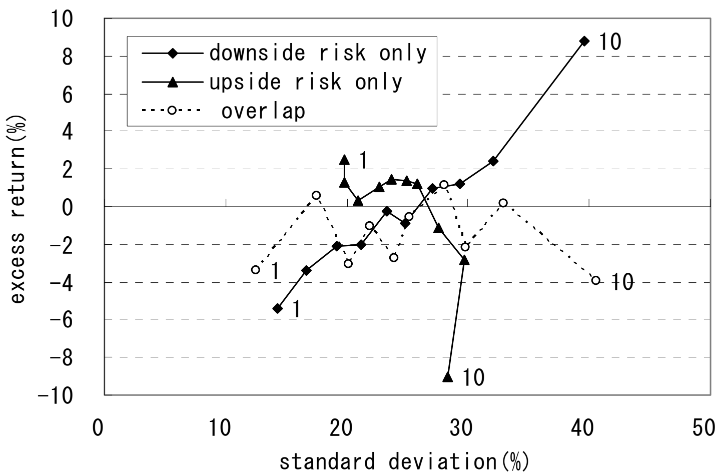


Exhibit 7 Summary of Rank Portfolio Analysis (whole period)

	volatility	Downside risk	Upside risk	mean return	Downside risk only	Upside risk only	Overlap
Regression coefficient	−0.06	0.25	−0.27	1.34	0.50	−0.65	−0.01
t value	−0.65	8.78	−2.30	3.17	12.41	−2.61	−0.09
10−1(return differences %)	−2.13	6.21	−5.68	18.59	14.20	−11.54	−0.60
10−1(risk differences %)	22.52	26.99	18.77	9.68	25.31	8.55	28.13
the continuity of risk	◎	◎	◎	×	◎	○	◎

・the continuity of risk : ◎ established, ○almost established, × not established

Interrelationship of three risk criteria

From the above argument, the possibility that the volatility effect reflected downward risk and upward risk arose. To confirm this possibility, we analyzed the correlation and intersectional ranking portfolio methods.

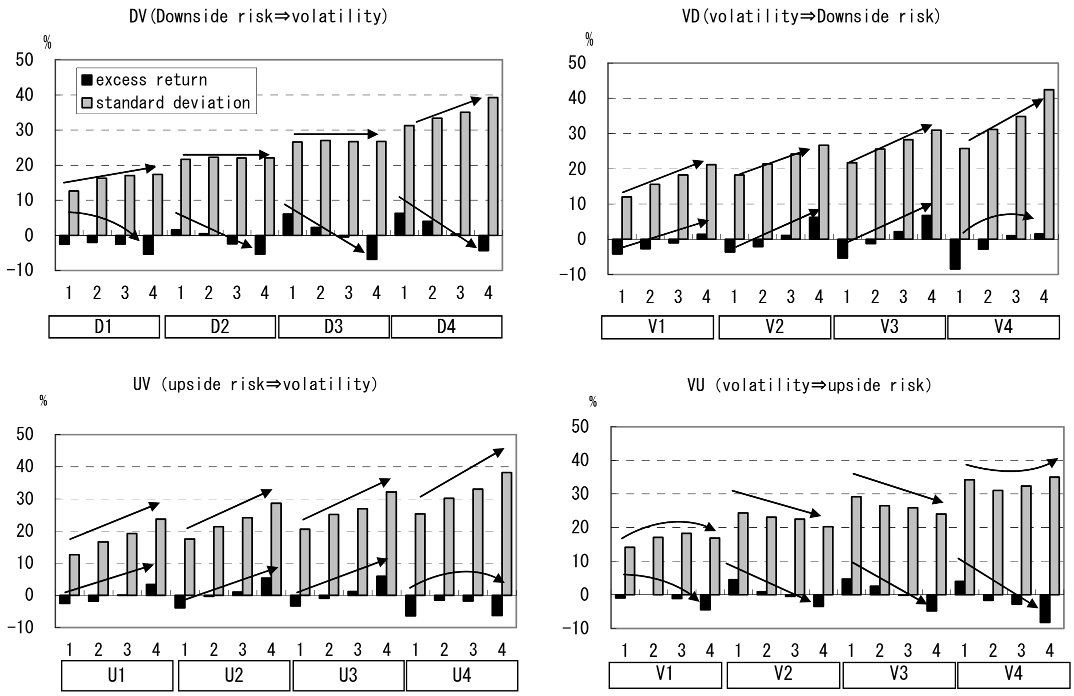
In correlation analysis, we examined cross-sectional ranking correlation with volatility, downside risk, and upside risk. If the ranking correlation coefficient is high, stocks in the portfolio should be similar.

From January 1990 to December 2010, the mean of the ranking correlation between volatility and downside risk was 0.90, volatility and upside risk 0.97, and downside risk and upside risk 0.78. Therefore, upside risk is more likely to have a big influence on the return characteristics of the volatility ranked portfolio because the correlation of volatility and upside risk is extremely high.

In intersectional rank portfolio analysis, we can examine the difference in the influence of one risk when the other risk is at the same level. We divided stocks into four rankings for volatility, and we divided each rank into four for downside risk (VD division). V means volatility, D downside risk, U upside risk. The reverse division transcribes DV division. Because we examine with three risk criteria, we perform the exercise six times.

Exhibit 5 provides intersectional rank portfolio analysis (we left it out, but DU division is similar to DV division and UD is similar to VD). With volatility level equal (VD, VU), when downside risks increase, both return and standard deviation increase, and when upside risks increase, return decreases and standard deviation levels off or even decreases. On the other hand, volatility reflects the basic tendency of downside risk (return decreases and standard deviation levels off) for DV division, and the basic tendency of upside risk (both return and standard deviation increase) for UV division (both divisions show a characteristic opposite to each first divided risk). Therefore, downside risk and upside risk each have a peculiar characteristic, and the characteristic which is called volatility effect is more likely to simply be the combination of these two risks. In other words, concerning subsequent return, we can interpret that volatility level is not important, but quantity including downward risk and upward risk is.

Exhibit 8　Intersectional Rank Portfolio Analysis (Whole period)



4. Conclusion

We examined the relation between three risk criteria (downside risk, upside risk, and volatility) and subsequent return (risk premium). As a result, we found that investors behave asymmetrically for negative return (downside risk) and positive return (upside risk), which seemed to be suggested by prospect theory. The trade-off between downside risk and realized return is established, while reverse trade-off exists for upside risk. This tendency became clearer when purifying downside and upside risk.

In terms of target return for downside and upside risk, the zero target one has better properties than the mean standard. With zero target return, we assume negative return a downward risk and positive return an upward risk, which is easy to recognize intuitively, and accessibility is probably high. Furthermore, comparing to mean or relative, zero return is the most consistent with loss aversion under prospect theory. There are a lot of articles on mean target analysis in previous studies of downside risk.

Because mean target downside risk characteristics resemble volatility risk characteristics, it is difficult to distinguish volatility from downside risk.

The explanation from the viewpoint of behavioral theory regarding the result of this paper is as follows. Investors (bounded rational) require a higher premium for stocks with high downside risk. Specifically, they do not purchase them when the price does not fall enough. On the other hand, while such investors (bounded rational) calmly invest in stocks with high upside risk, they will pay relief charges. Specifically, they overestimate stocks with high upside risk and purchase at a high price. As for return reversal, it can be interpreted as a composition effect of downside risk and upside risk from the relationship of mean return ≒ upside risk - downside risk.

Examining the interrelationship of three risk criteria, the ranking correlation between volatility and upside risk is 0.97, extremely high. Therefore, upside risk is more likely to have a substantial influence on volatility effect as stocks with higher volatility earn higher returns. Through the intersectional ranking portfolio analysis of three risk criteria, both downside and upside risk have peculiar characteristics, and, the volatility effect is more likely to be just the combination of these two risks. Thus, using a framework of loss aversion and asymmetry between profits and losses for financial analysis is thought to be effective. Furthermore, a stock return generation process that adopts path dependence deserves examination.

Considering the direction of future research, because the momentum effect that is not confirmed in Japan exists in other markets, different relationships for downside risk, upside risk, and subsequent return are expected. And, we think these effects are closely related to a period which recognized losses, namely the past 60 months for this paper.

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