1. Introduction

The liquidity of government bonds and other flow-based financial products is generally defined as the ease with which those products are bought and sold in the markets. The bid-ask spreads\(^1\) are used as a variable to express the liquidity quantitatively, and the liquidity is often discussed on the basis of an analysis of bid-ask spreads. The larger the spreads, the lower the liquidity, and vice versa. For bonds, however, the significance of bid-ask-spreads is less clear.

For example, these spreads may be defined as the differences of yields (rates\(^2\)) between ask prices and bid prices in the interdealer market. This is the natural way of thinking in light of stock

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\(^1\) In practical bond market business, "offer-bid spreads" are often used, but in this paper, we used "bid-ask spreads" for consistency with analyses and previous studies of other markets.

\(^2\) In Japan, many bond transactions are conducted on the basis of final yields by simple interest calculation, which are called "rates" in this paper.
markets, etc. where most of transactions take place at exchanges, and there is also a previous study for Japanese government bonds (JGBs) which shows interesting results. In the bond markets, on the other hand, where over-the-counter (OTC) trading accounts for most of transactions, investors do not have access to the interdealer market but instead conduct negotiated transactions with their respective dealers. Recently, many bond dealers release reference rates as indications for these negotiated transactions through information vendors or on the Internet, but bid-ask spreads observed there are naturally different from those spreads on the interdealer market. A comparison shows that for benchmark issues that are traded frequently on the markets, bid-ask spreads in interdealer trading are generally smaller than those offered by individual dealers. However, since indicative rates are not always available for all outstanding issues in interdealer trading, other than some benchmark issues, actually traded rates may in some cases be more closely reflected by indicative rates collectively offered by individual bond dealers than indicative rates in the interdealer market.

Under these circumstances, it is not so easy to answer the question of what would result when an investor intending to buy or sell bonds asked dealers to offer their rate. Bond dealers do not necessarily present reference indicative rates of their own companies as they are, let alone indicative rates in interdealer trading. Furthermore, as rates offered vary between dealers, it is common that rates at which transactions are actually executed vary depending on how many dealers an investor asks to offer rates. In other words, the outcome also depends on an investor’s method of executing transactions, and roughly speaking, an investor cannot be sure about the outcome until he or she actually asks dealers to present their rates. In this paper, we analyzed rates offered by bond dealers on an electronic trading platform and estimated the structure of bid-ask spreads based on the distribution of offered rates. However, it is difficult to directly estimate the bid-ask spreads since it is rare that buying and selling of the same issues are conducted almost simultaneously even in electronic trading. Thus, in this paper, we derived the relationship between the distribution of offered rates and the bid-ask spreads by introducing a simple model. With the use of this model, we can examine the structure of bid-ask spreads as well as the structure of liquidity by analyzing the dispersion of offered rates.

In the second section, which follows this introductory section, we present and outline a simple model concerning the distribution of rates offered by dealers in bond transactions. We provide an overview of electronic trading data used in the analysis in the third section, and in the fourth section, we examine the liquidity structure of the JGB market and its time variance based on the actual results of analysis.

The fifth section examines the relationship between the liquidity of the JGB market and
execution cost. In particular, we considered the effect of asking multiple dealers to present rates.

In the sixth section, which summarizes the discussions in this paper, this author presents his views about the liquidity of the JGB market and execution cost.

2. Bond Trading Inquiry Model

The bulk of bond transactions is conducted on the OTC market. In an ordinary transaction, an investor asks a bond dealer (commonly, a securities company) to offer a trading rate, and if the offered rate is desirable, the investor concludes a trading contract, and if not, cancels the proposed transaction. This act to ask for the presentation of a rate is an “inquiry” for trading, and the act to offer a rate by a bond dealer is called “market-making.” These acts are usually conducted over the phone via a sales representative of a securities company, but in recent years, they are often carried out on an electronic trading platform or, as necessary, in combination with the use of e-mail.

Rates offered in response to an inquiry by an investor usually vary among bond dealers. To put it plainly, the offered rates are determined by dealers’ market-making abilities, or more specifically, by comprehensively reflecting their risk management abilities and positioning (the status of inventories) as well as the overall cost of bond dealing. Below, we describe the existence of the dispersion of offered rates among dealers, as a given, with a simple model. Only an overview of the model is provided below, with the mathematical portions of the model explained in the appendix at the end of this paper.

First, let us assume that n bond dealers are providing market-making services, and rates offered by respective dealers to sell orders by an investor (bid rates) are $BR_i \ (1 \leq i \leq n)$ and rates offered by respective dealers to buy orders by an investor (ask rates) are $AR_i \ (1 \leq i \leq n)$. Here, the maximum value of ask rate $AR_i$ is $BAR$ (the most advantageous ask rate for an investor), the minimum value of ask rate $AR_i$ is $WAR$ (the most disadvantageous ask rate), the minimum value of bid rate $BR_i$ is $BBR$ (the most advantageous bid rate), and the maximum value of bid rate $BR_i$ is $WBR$ (the most disadvantageous bid rate). It is also assumed that $AR_i$ is exponentially distributed below $M-D$ and $BR_i$ is exponentially distributed above $M+D$, and the distribution function is the same for all dealers. Here, $M$ stands for the prevailing market rate (median rate) in an ideal market condition, such as negligible cost, while $D$ is the marginal value of the divergence of rates offered by bond dealers from the median rate. In other words, $D$ is the parameter that shows the extent to which the best rate comes close to the median rate $M$ when the number of dealers for rate inquiries is increased. Figure 1 shows the basic concept of this model when the number of dealers is 5 ($n=5$).

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4 They are the abbreviation for “Best Ask Rate,” “Worst Ask Rate,” “Best Bid Rate” and “Worst Bid Rate,” respectively.
5 This assumes the homogeneity (uniformity) of bond dealers.
6 Intuitively, $D$ may be considered to have a value close to zero, but this paper is not intended to verify that.
Here, the expected value of the difference between the best rate and the worst rate is defined as the “rate dispersion” and expressed as $V$. In other words, $V$ should be $WBR - BBR$ when an investor is making an inquiry to sell, and $BAR - WAR$ when an investor is making an inquiry to buy. The expected value of the bid-ask spread for each dealer is called the “individual spread” and expressed as $S$. Here, the relationship between the rate dispersion $V$ and $S$ is expressed as follows (Appendix (2) Formula (9)):

$$V = \frac{1}{2} \sum_{k=1}^{n-1} \frac{1}{k} (S-2D) \quad (1)$$

In particular, the rate dispersion $V_5$ in the case of five dealers for rate inquiries is expressed as follows (Appendix (2) Formula (10)):

$$V_5 = \frac{25}{24} (S-2D) \quad (2)$$

In other words, as individual spreads become larger, the rate dispersion also grows larger accordingly. In the case of five dealers for rate inquiries, the expansion in individual spreads almost directly translates into the margin of expansion in the rate dispersion$^7$. Using this relationship, it becomes possible to estimate individual spreads, or changes in average bid-ask spreads in the market and their structure by observing the rate dispersion. In the following section onward, we estimate the rate dispersion $V_5$ in the case of five dealers for rate inquiries from data observed on the electronic trading platform, and based on the estimate, consider the liquidity of the JGB market and execution cost.

### 3. Data for Analysis

The analysis in this paper used trading data for the platform operated by Yensai.com Co., Ltd. (hereinafter “Yensai”). For an overview of Yensai, please see Appendix (3). In brief, it is an electronic trading platform for bonds that allows investors to simultaneously make rate inquiries to up to five participating securities companies and conduct transactions with the securities company that offered the most desirable rates.

The analysis covered data of transactions covering nine years from 2003 to 2011 for which requests for rates were made to five bond dealers and all of them offered rates. They include data of proposed transactions canceled without reaching trade agreements for reasons such as the offered rates falling short of investors’ expectations. The analysis also excludes a small number of data items that do not reflect actual market conditions.

---

$^7$ On the assumption of up to $D=0$, the rate dispersion $V_5$ in the case of five dealers for rate inquiries and individual spreads $S$ are almost identical in their levels.
Table 2 Overall Picture of Data for Analysis and the Rate Dispersion

![Table Image](image)

Note 1: This figure shows the rate dispersion and the number of data by the date of proposed transactions (by the year, in the direction of columns) and the period remaining to maturity for issues covered (in years, in the direction of rows).

The upper row of each section indicates the rate dispersion (Unit: bp=0.01%) and the lower row the number of data. The number of data is zero for 31-35 years remaining to maturity and for blank space in the figure.

Note 2: The period remaining to maturity is based on the date of transaction. Value in the section for the period of x years remaining to maturity includes all data for less than x.125 years remaining to maturity.

Note 3: It should be noted that government bonds with different maturities at issue are included in sectors for the same period remaining to maturity.

Note 4: Trading in 30-year government bonds and 40-year government bonds on Yensai started in July 2004 and April 2009, respectively.

Source: Yensai.com

as dealers apparently made errors in their offered rates. As a result, the data for analysis covered 117,360 transactions. We computed the rate dispersion, or the difference between the best and worst offered rates, for each date of transaction by the data for the period of 10 years remaining to maturity (by years) of issues covered, the average value of the rate dispersion is shown in Figure 2.

This shows that the number of data increased with the spread of electronic trading, and grew particularly significantly after 2007. By period remaining to maturity, the number of proposed transactions was the largest for the two-year, five-year and 10-year sectors, and it is also apparent that the number of transactions in the 20-year and 30-year sectors was larger than for the surrounding periods. This is evidently because new bonds are issued for these maturities on a regular basis.

4. Results of Analysis and Consideration

In this section, we focus on and discuss structures characteristic of data shown in the preceding section. Here, we considered the rate dispersion computed on the basis of the data for transaction inquiries, which, because of the
relationship expressed by Formula (2), directly reflects changes and structural characteristics of bid-ask spreads. In other words, within the framework of the analysis of this paper, the consideration of the rate dispersion is presumed to be tantamount to the consideration of bid-ask spreads and liquidity.

(1) Time Change in the Rate Dispersion

As is clear from Figure 2, the rate dispersion changes over time. For example, in 2008, when we witnessed the major disarray in the global financial and capital markets as symbolized by the Lehman Shock, the rate dispersion was very high for any period remaining to maturity, but the rate dispersion gradually dwindled in the following three years.

In order to examine the time change in the rate dispersion in greater detail, Figure 3 shows the monthly rate dispersion for the two sectors of “five-year JGBs with five years remaining to maturity” and “10-year JGBs with 10 years remaining to maturity,” that have a relatively large number of data. It can be easily imagined that the rate dispersion and bid-ask spreads are affected by market volatility. In order to ascertain this, Figure 3 also shows the monthly margin of change in yields (= the highest monthly traded yield – the lowest monthly traded yield) for the current issues of both sectors in interdealer trading via Japan Bond Trading Co.

This figure indicates that there exists a strong correlation between the rate dispersion computed from data for proposed transactions on the electronic platform and the market volatility computed on the basis of contracted yields in interdealer trading. This apparently reflects that increased market volatility led to an expansion in bid-ask spreads, which in turn translated into the wider rate dispersions.

**Figure 3 Time Change in the Rate Dispersion and Market Volatility**

![Graph showing time change in rate dispersion and market volatility](image)

Note: The monthly range of yield fluctuation is the difference between the highest monthly yield and the lowest monthly yield for the current five-year and 10-year bonds (the latest issues) in interdealer trading via Japan Bond Trading Co. For reference, the month-end final yield in interdealer trading is also shown.

Source: Japan Bond Trading, Yensai.com

Until May 2003, the rate dispersion stayed very small, as market volatility declined to extremely low levels in the unprecedentedly low interest rate climate where long-term interest rates dipped below 0.5%. However, long-term interest rates turned upward in June 2003, and as the market entered a phase of high volatility subsequently due to the so-called “VaR (value-at-risk) shock,” the rate dispersion also increased significantly in tandem. In September 2003, when banks were said to have increased sales of mainly medium-term government bonds due to VaR-induced tougher constraints on risk-taking, yields of five-year government bonds topped 1%
at one point, and reflecting this development, the rate dispersion also rose to around 1.2bp.

This phase of market fluctuations around the VaR shock came to an end in a relatively short span of time, and the market volatility and the rate dispersion declined until the first half of 2005. From the second half of 2005 through the first half of 2006, however, both the market volatility and the rate dispersion rose in some phases amid speculation about a possible shift in monetary policy, including the lifting of quantitative easing. Furthermore, from the second half of 2007 through the first half of 2008, the rate dispersion expanded significantly in the wake of the financial market turmoil that can be traced to an escalation of the subprime loan problem and then to the Lehman Shock.

Then, in 2010-2011, these shocks calmed down and the market volatility declined in reaction. Reflecting the stability of long-term interest rates at lower levels (the stability of bond prices at higher levels), the rate dispersion stood at around 0.4bp for government bonds with 10 years remaining to maturity, the lowest levels for the duration of the analysis.

(2) Term Structure of the Rate Dispersion

Figure 2 suggests that the rate dispersion is varied for each period remaining to maturity, and hence it has the term structure. It is considered that this reflects that liquidity is varied for government bonds in each period remaining to maturity.

Figure 4 shows the rate dispersion in graph form in each of the six trading years from 2006 through 2011, with periods remaining to maturity for issues traded on the horizontal axis in years and the rate dispersion for corresponding periods remaining to maturity on the vertical axis. However, bonds with the same periods remaining to maturity but with different initial maturities at issuance are shown separately. The graphs indicate that though levels of the rate dispersions are different depending on the years when bonds were traded, there are common characteristics between the periods remaining to maturity and the rate dispersions.

Figure 4 Term Structure of the Rate Dispersion

Note: Data used in Figure 2 are further subdivided by initial maturities at issuance and shown in graph form for 2006-2011. But 20-year government bonds with nine years or less remaining to maturity are not shown for the easier display of
the graphs as the rate dispersion for them is large due to a small number of data.
Source: Yensai.com

Specifically, for all two-year, five-year, 10-year, 20-year, 30-year and 40-year government bonds, the rate dispersions are small for issues for which only a short amount of time has passed since issuance. Comparison of the current issues with the longest periods remaining to maturity (for example, two-year government bonds with two years remaining to maturity and five-year government bonds with five years remaining to maturity) shows that the rate dispersions are small for five-year and 10-year government bonds and tend to grow larger for two-year, 20-year, 30-year and 40-year government bonds, in that order.

This pattern can be construed to indicate that the liquidity of government bonds declines as volumes traded on the market drop in tandem with the passage of time from issuance, resulting in an expansion of the rate dispersions. The liquidity immediately after issuance is high for 10-year government bonds, a benchmark for long-term interest rates, and five-year government bonds, which have the central role among medium-term maturities. For ultralong-term government bonds, the liquidity declines for 20-year, 30-year and 40-year government bonds, in that order. These structures are consistent with our practical senses in the JGB market.

(3) The Rate Dispersions for Ultralong-Term JGBs

Figure 4 shows that the rate dispersions for ultralong-term government bonds have narrowed significantly in recent years. The trends over the past six years shown in these graphs indicate that the narrowing of the rate dispersion for 30-year government bonds is particularly remarkable. The average rate dispersion in 2008 was 0.67bp for 10-year bonds, 0.97bp for 20-year bonds and 1.23bp for 30-year bonds, but the average rate dispersion in 2011 stood at 0.36bp for 10-year bonds, 0.54bp for 20-year bonds and 0.57bp for 30-year bonds, indicating that the wide rate dispersion for 30-year bonds relative to 20-year bonds has all but disappeared. This means that the liquidity of 30-year government bonds has improved significantly to come much closer to that of 20-year government bonds.

As for ultra-long-term government bonds, a significant narrowing of rate dispersions has been observed throughout the years shown in Figure 5 not only for periods to maturity close to the issuing maturities such as 20 years and 30 years but also for periods to maturity far from the issuing maturities such as 11-19 years and 21-29 years. It is highly likely that this has resulted from the Finance Ministry’s efforts to intensify bond auctions for enhanced liquidity. Auctions for enhanced liquidity, launched in April 2006, were expanded to two monthly auctions for ¥300 billion each in July 2009, with one of the two involving the reopening of outstanding 20-year and 30-year government bonds with 15-29 years remaining to maturity. These auctions for enhanced liquidity
are thought to have had a significant impact in enhancing the liquidity of ultralong-term government bonds as a whole.

It is believed that efforts to nurture the market by regular issuance and larger issuance lots as well as efforts to further develop the market through auctions for enhanced liquidity have contributed to the narrowing of the rate dispersions for ultralong-term government bonds.

(4) Amount of Proposed Transactions and the Rate Dispersion

The relationship between amounts of proposed transactions and the rate dispersions are very important in considering execution costs, including impacts on the market. In order to examine the relationship, we divided the amounts of proposed transactions into the five categories of (1) less than ¥100 million; (2) ¥100 million to less than ¥1 billion; (3) ¥1 billion to less than ¥5 billion; (4) ¥5 billion to less than ¥10 billion; and (5) ¥10 billion or more, and showed the rate dispersions for the two sectors of “five-year JGBs with five years remaining to maturity” and “10-year JGBs with 10 years remaining to maturity” in Figure 5.

For the relationship between amounts of proposed transactions and the rate dispersions, it is generally assumed that the larger the amount of proposed transactions, the larger the market impact is and the larger the rate dispersion grows. However, Figure 5 indicates that there is almost no relationship between the two and that the rate dispersion occasionally grows rather larger when the amount of proposed transactions is ¥1 billion or less. This differs from the general assumption but causes no sense of strangeness in practical terms on the JGB market. Bond dealers prefer transactions in reasonably large amounts to those in very small amounts relative to the market size. They are thought to more offer more aggressive rates in response to inquiries for transactions in reasonably large amounts.

**Figure 5 Amounts of Proposed Transactions and the Rate Dispersion**

In looking into the market impact of amounts of proposed transactions, it becomes necessary to make an analysis of data for inquiries for transactions in even larger amounts. Unfortunately, however, the number of items of data for inquiries for such transactions is not sufficient at this point. It is hoped that a further accumulation of data on electronic trading may make such analysis possible.
5. Inquiries at Multiple Dealers and Execution Cost

Thus far, we have looked at the liquidity structure of the JGB market by using the rate dispersion in the case of rate inquiries made to multiple bond dealers. Now, we could like to add some comments about execution cost. Here, execution cost is narrowly interpreted and is almost synonymous with the divergence from the median rate.

As discussed in the first section, prices or yields with which investors can trade on the JGB market are not determined objectively and uniquely. They vary depending on to which bond dealers or to how many bond dealers investors make rate inquiries. When the homogeneity of bond dealers is assumed, as in the model of this paper, the number of bond dealers to which rate inquiries are made is of particular importance.

In the model used in this paper, the divergence of the best rate from the marginal value is inversely related to the number of bond dealers under a certain range of bid-ask spreads (or under a certain range of rate dispersions) (Appendix (2) Formula (11)). This relationship is shown in graph form in Figure 6. With the number of dealers for rate inquiries on the horizontal axis and the divergence of the best rate from the marginal value (M+D for bid, and M-D for ask) on the vertical axis, this figure shows three scenarios where the expected rate dispersion \( V_5 \) in the case of five dealers for rate inquiries is 0.5bp, 1bp and 2bp, respectively. Based on the above discussions, the scenario of \( V_5 = 0.5 \text{bp} \) corresponds to the highest liquidity situation (the smallest bid-ask spread) and the scenario of \( V_5 = 2 \text{bp} \) to the lowest liquidity situation (the largest bid-ask spread). In terms of specific market phases, \( V_5 = 0.5 \text{bp} \) is close to the state of liquidity for the five-year and 10-year sectors in 2010-2011, while \( V_5 = 2 \text{bp} \) is a level somewhat higher than the state of liquidity for the 20-year sector around the Lehman Shock.

![Figure 6 Number of Dealers for Rate Inquiries and Execution Cost](image)

Figure 6 indicates that when the number of dealers for rate inquiries is increased, the narrowly-defined execution cost, or the divergence of the execution rate from the median rate, becomes smaller. In other words, this means that for investors, the larger the number of dealers for rate inquiries, the broader the opportunities for them to execute transactions at more advantageous rates. This is all but obvious, even without bringing out the model. On the other hand, the larger number of dealers for rate
inquiries also requires more time, and the longer time required between the commencement and completion of transactions is likely to increase the broadly-defined execution cost, including possible execution errors. Furthermore, as can be seen from Figure 6, the effect of rate improvements for investors dwindles in tandem with the increase in the number of dealers for rate inquiries. Thus, depending on constraining conditions, inquiry methods and market phases, the optimum number of dealers for rate inquiries and expected rates offered could change. In light of this, “three bond dealers for rate inquiries,” which is the standard guideline for investment management institutions in Japan, appears to be the reasonable criteria.

What should be noted, however, is that the margin of reduction in the execution cost resulting from an increase in the number of dealers for rate inquiries is also affected by the rate dispersion. More specifically, the larger the rate dispersion is (the lower the liquidity is), the larger the margin of rate improvements resulting from an increase in the number of dealers for rate inquiries. Specifically, in Figure 6, when the number of dealers for rate inquiries is increased from one to three, while the expected margin of rate improvements is approximately 0.16bp in the scenario of $V_5 = 0.5bp$, a rate improvement of approximately 0.64bp can be expected in the scenario of $V_5 = 2bp$. When the number of dealers for rate inquiries is increased from one to five, the expected rate improvement is approximately 0.19bp and 0.77bp, respectively.

In other words, while the cost of executing transactions in lower-liquidity issues or under lower-liquidity market conditions is greater than that for higher-liquidity issues or under higher-liquidity conditions, the difference can be partially improved by an increase in the number of dealers for rate inquiries. More specifically, for transactions in market phases with high volatility, in issues for which many years have passed since issuance and in issues with smaller outstanding amounts, the benefit of increasing the number of dealers for rate inquiries is relatively large.

6. Conclusion

Amid the remarkable growth of the size of the JGB market in recent years, the market liquidity is also said to have been enhanced significantly. That was confirmed by the analysis of trading inquiry data for electronic trading in this paper. The enhancement of the market liquidity is believed to have benefited investors by reducing execution cost.

On the other hand, the results of analysis shown in Figure 3 suggest that the recent enhancement of liquidity can also be traced, to a great extent, to the decline in market volatility. Amid the continuation of the record-setting prolongation of the range-bound JGB market, bond dealers are engaged in more intense competition in the rates they offer, and this also appears to bolstering the ostensible liquidity. Under such market conditions, it can be assumed that the differences in rates offered by bond
dealers became narrower and, what’s more, the cost of taking time to make transaction inquiries was small. In other words, it is believed that execution methods did not affect execution cost so much. Needless to say, however, there is no certainty as to how long such market conditions will last. The analysis in this paper suggests that if the market volatility increases, that would be reflected directly in execution cost.

When the JGB market showed violent swings around the “Trust Fund Bureau Shock” from late 1998 to early 1999, an event many current market participants have not experienced directly, we witnessed a number of phases where it appeared that the market liquidity had been lost completely. Tracing back further, we have the “history” of market phases of lost liquidity, such as the tumble of “roku-ichi” government bonds (which means 6.1% coupon JGBs) around 1980 and the bursting of the bond market bubble in 1987. On such occasions, investors presumably had to bear huge execution costs, and above all, the debacle of the JGB market had a significant impact on the overall financial and capital markets. No optimism is warranted by ruling it out as an “unexpected event.”

If we cannot rule out an increase in market volatility that could lead to higher interest rates in the future, at least as a possibility, it is necessary to develop a robust trading infrastructure that can withstand the shock. If we are to consider countermeasures in an extension of the discussions in this paper, it is important to build up a mechanism that allows investors to make trading inquiries to many dealers smoothly in a short period of time. Such mechanism can be expected to allow investors to find rates more advantageous to them in a short period of time and the overall market to facilitate the appropriate distribution of interest rate risks. Realistically, however, it is necessary to make an objective and quantitative analysis of the market structure before moving to that stage. Currently, the Japanese bond market still lags behind other markets in terms of the utilization of information technology and the analysis of transaction information. We need to deal with these problems first.

Appendix

(1) Distribution of the Maximum Value and the Minimum value

Generally speaking, when a variable p falls into line with the probability density function f(p) and the cumulative distribution function F(p), of the n number of samples, p_1, p_2, p_3, \ldots, p_n, the maximum value is set as x, the minimum value as y, and the difference between x and y as z.

\[
\begin{align*}
x &= \max(p_1, p_2, p_3, \ldots, p_n) \\
y &= \min(p_1, p_2, p_3, \ldots, p_n) \\
z &= x - y
\end{align*}
\]

Here, when the cumulative distribution function of distributions into which x, y and z fall is set as U(x), V(y) and W(z), respectively, they can be expressed as follows:
Theoretical value (uniform distribution)

Further, when the probability density function is set as \( u(y) \), \( v(z) \) and \( w(r) \), respectively, they can be obtained as below by differentiating the above formulas:

\[
\begin{align*}
  u(y) &= n\{F(y)\}^{n-1} f(x) \\
v(z) &= n\{1-F(z)\}^{n-1} f(x) \\
z(r) &= n(n-1)\int\{F(x+r)-F(x)\} f(x)dx
\end{align*}
\]  

Figure 7 shows the results of actual calculations of them for the exponential distribution and the uniform distribution, with their respective expected value also shown.

Figure 7 Distribution of the Maximum Value and the Minimum Value in the Exponential Distribution and the Uniform Distribution

<table>
<thead>
<tr>
<th></th>
<th>Probability Density Function</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>( f(p) = (1/a) \exp(-p/a) )</td>
<td>a</td>
</tr>
<tr>
<td>Max. value</td>
<td>( u(x) = n(a) {1-\exp(-x/a)}^{n-1} )</td>
<td>( a^{n-1} )</td>
</tr>
<tr>
<td>Min. value</td>
<td>( v(y) = 1-\exp(-ny/a) )</td>
<td>( a/n )</td>
</tr>
<tr>
<td>Max. value</td>
<td>( w(z) = n(n-1){\exp(-z/a)}^{n-2} )</td>
<td>( a^{n-1} )</td>
</tr>
<tr>
<td>Min. value</td>
<td>( z(r) = n(n-1)b{z/b}^{n-2}{z/b}^{n-1} )</td>
<td>( b(n+1) )</td>
</tr>
</tbody>
</table>

(2) Mathematical Supplementation of the Trading Inquiry Model

In the analysis of this paper, an ask rate AR\(_i\) and a bid rate BR\(_i\) offered by individual bond dealers are to fall into line with the exponential distribution, and the probability density function of their respective distributions, \( f(\cdot) \) and \( g(\cdot) \), are to be expressed as follows:

\[
\begin{align*}
  f(BR) &= \frac{1}{a} \exp[(BR-M-D)/a] \\
g(AR) &= \frac{1}{a} \exp[(M-D-AR)/a]
\end{align*}
\]  

Here, \( a \) is the parameter that shows the extent of dispersion, and the larger the value of \( a \) is, the larger the dispersion is of rates offered by each bond dealer or rates offered by individual bond dealers for each trading inquiry.

Figure 8 shows the distribution of the rate dispersion computed from data for actual trading inquiries for 10-year government bonds with 10 years remaining to maturity in 2011, together with theoretical values of the exponential distribution and uniform distribution models. For each distribution, there is only one parameter, determined to produce the identical average value. The graph indicates that the exponential distribution produces a good fitting. Based on this result, this paper adopted the exponential distribution model in the form of Formula (4).

Figure 8 Observed Value and Model Value of the Rate Dispersions

Source: Yensai.com
Now, when offered rates fall into line with this model, the expected values of bid and ask rates offered by a bond dealer in response to an investor’s request are simply calculated as follows (see Figure 7). Here, \( E[\cdot] \) is the expected value operator:

\[
\begin{align*}
E[BR_i] &= M + D + a \\
E[AR_i] &= M - D - a
\end{align*}
\]  

(5)

Here, the bid-ask spread (individual spread) \( S \) is as follows, when this investor made a rate inquiry to one bond dealer:

\[
S = E[BR_i - AR_i] = 2D + 2a
\]  

(6)

In this case, the expected value of the divergence of the four best and worst rates from the median rates can be obtained as follows, based on the distribution of the maximum value and the minimum value of the probability density function that falls into line with the exponential distribution (Appendix (1) Figure 7):

\[
\begin{align*}
E[BBR-M-D] &= E[M-D-BAR] = \frac{a}{n} \\
E[WBR-M-D] &= E[M-D-WAR] = \sum_{k=1}^{n} \frac{a}{k}
\end{align*}
\]  

(7)

Thus, under this model, regardless of whether an investor is making an inquiry to sell or an inquiry to buy, the expected value of the rate dispersion, \( V = E[BAR - WAR] = E[WBR - BBR] \), is as follows:

\[
V = \frac{1}{2} \left( \sum_{k=1}^{n-1} \frac{a}{k} \right)(S - 2D)
\]  

(8)

If \( a \) is eliminated from Formula (6) and Formula (8), we can obtain the following:

\[
V = \frac{1}{2} \left( \sum_{k=1}^{n-1} \frac{1}{k} \right)(S - 2D)
\]  

(9)

In the case of five dealers for rate inquiries (\( n=5 \)), we can obtain the following:

\[
V_5 = \frac{25}{24} (S - 2D)
\]  

(10)

Further, based on Formula (6), Formula (7) and Formula (10), the expected value of the divergence of the best rate from the median rate can be expressed as follows:

\[
E[BBR-M-D] = E[M-D-BAR] = \frac{12}{25n} V_5
\]  

(11)

(3) Yensai Platform

Yensai is a platform for broking transactions in JGBs. As of the end of June 2012, there are a total of 14 securities companies that make markets for JGBs on the platform.

Subject to broking on the platform as of the end of June 2012 are a total of 308 issues of interest-bearing 2-year, 5-year, 10-year, 20-year, 30-year and 40-year JGBs. Investors make trading inquiries for these JGBs by entering or choosing (1) specific issue; (2) sell or buy; (3) amount of proposed transaction (a minimum amount of ¥10 million, in units of ¥50,000, limited to tax-exempt book-entry issues); (4) delivery date (a date on or after the standard delivery date that does not fall under the category of transactions with delayed settlements); and (5) securities companies asked to offer rates (up to five companies). Figure 9 displays the virtual trading request screen on the Yensai platform, showing an investor making rate inquiries to the
five securities companies of J, D, K, B and H to “buy ¥10 billion worth of the 319th issue of 10-year JGBs for delivery on February 21.”

Figure 9 Yensai Screen of Rate Inquiry for Proposed Transaction (Reference)

Securities companies that receive inquiries are supposed to offer rates within the time limit (20 seconds). Rates should be offered in units (minimum unit) of 0.01bp (0.0001%). Securities companies set the time of validity for their offered rates, and when trading orders are not placed during that time, they can cancel the initially offered rates and offer new rates. An investor can place a trading order when at least one securities company offers its rate, but usually waits until all the securities companies to which rate inquiries were made offer their rates and places a trading order with the securities company that came up with the most advantageous rate. The investor may cancel proposed transactions when the rates offered by all the securities companies fall short of the investor’s expectations.

References

The author made the analysis in this paper with data provided by Yensai.com Co., Ltd., but the views expounded in this paper are the personal views of the author and are not the views of Yensai.com Co., Ltd. or any other organizations the author belongs to.