Market Portfolio, Benchmark Indices, and Universal Hedge Ratio

Nomura Securities Financial Research Center

Tomoyuki Ohta

Abstract

Market-capitalized indices are extensively used as a benchmark for pension funds in Japan. The rationale behind this may be because it is thought that the market portfolio is optimal. However a market-capitalized portfolio is not the market portfolio. In other words, a market-capitalized portfolio is not necessarily optimal for each investor. It may be better to use customized benchmark indices for domestic bonds, domestic equities, foreign bonds, and foreign equities, instead of the market-capitalized portfolio. In particular for domestic bonds, use of a customized index as a benchmark is recommended. Furthermore the universal hedge ratio (Black (1990)) may not be optimal for each investor.

1. Introduction

The use of benchmark indices has recently become popular for pension funds investment in Japan. In many cases, for every asset class (domestic bonds, domestic equities, foreign bonds, and foreign equities) performance indices based on market capitalization are used as benchmark indices. For example, pension funds in Japan usually use the NOMURA-BPI (NOMURA Bond Performance Index; formerly NRI-BPI)[1] for Japanese yen denominated domestic bonds, TOPIX for domestic equities, the Solomon-Smith Barney index for foreign bonds, and the MSCI index for foreign equities.

The theoretical reasoning behind such usage of benchmark indices may be that the market portfolio is optimal from the viewpoint of mean-variance analysis. Namely, if there are a number of risk assets, the relation between the volatility of asset return (standard deviation)

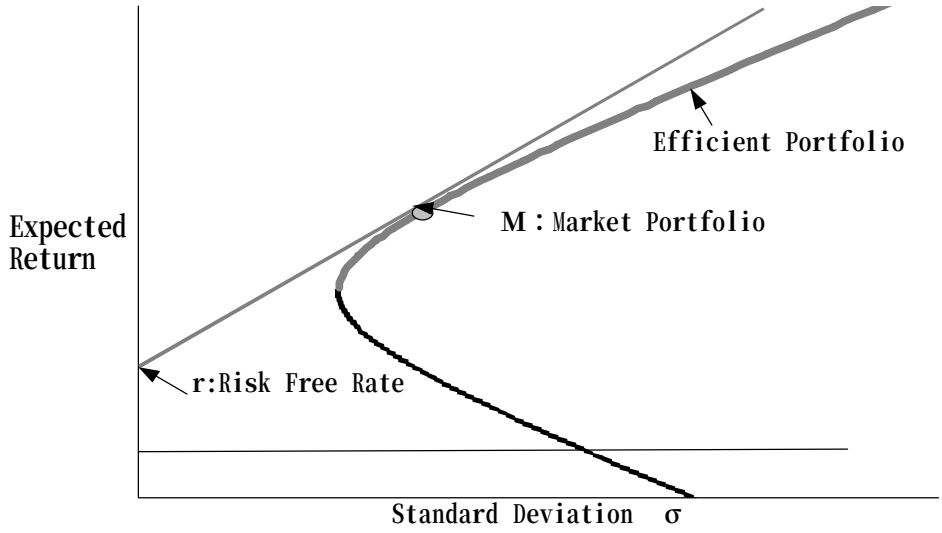
[1] NOMURA-BPI is now calculated and published by Nomura Securities Financial Research Center rather than Nomura Research Institute (NRI).

and expected return will be as shown in Figure 1. If there is a risk-free asset (the interest rate is r), its combination with some portfolio of risk assets, shown in Figure 1 as portfolio M, has higher expected returns compared to any other combination of risk assets with the same volatility. Therefore, every investor will select a portfolio in proportion to M as his (or her) portfolio of risk assets. Since every investor will hold a portfolio in proportion to M, portfolio M should be the market portfolio of risk assets. That means the market portfolio should be the optimal risk asset for every investor. So it may be reasonable to make the market-capitalized index as the market portfolio, and to use it for the benchmark.

However, if this implication implied by mean-variance analysis is correct, why is it necessary to make a decision of the allocation of asset classes such as domestic bonds, domestic equities, foreign bonds, and foreign equities? Is it not enough to have one universal bond-equity index for every investor? If a market-capitalized portfolio of assets is optimal, why do we not use a combined market-capitalized worldwide index? Shouldn't this be optimal for all investors?

But, in practice, such a worldwide bond-equity index is never used. Ordinarily, investors decide allocation of asset classes, and it is usually different from the market-capitalized weight of all asset classes. Are such investment decisions rational? This article discusses about the problems related to the market portfolio and benchmark indices.

Figure 1 Mean-Variance Analysis



2. Domestic Bonds

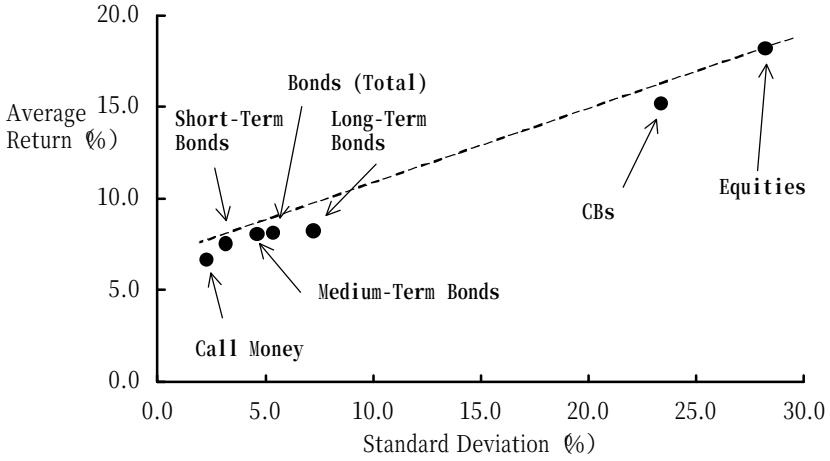
Since dealing with foreign-currency denominated assets is rather complex because of foreign exchange risk, we would like to discuss about domestic bonds and equities at first[2].

(1) Bond Portfolios

In practice, the problem with the market portfolio of bonds comes from the historical rate of return and volatility of the market-capitalized bond portfolio.

Figure 2 shows the standard deviation of rates of return and average return of various assets. When a straight line connecting the interest rate of short-term assets (e.g. call money) and that of equities is drawn, the average return of the bond portfolio (Bonds (Total) in Figure 2) may be below the line. If this relationship holds, a combination of equities and short-term assets will achieve higher average returns than bonds with similar volatility. In other words, the meaning of investment in bonds is suspicious.

Figure 2 Average Return and Standard Deviation of Each Asset
(1971～1990)

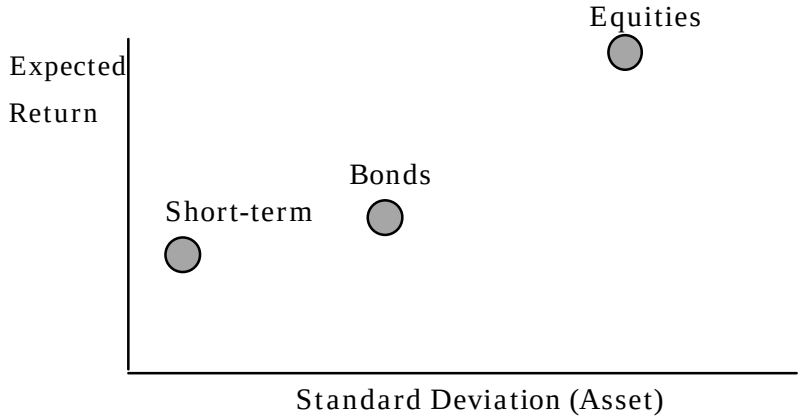


Sources: TOPIX with dividends (Nomura Research Institute), NOMURA-BPI, NOMURA-CBPI (Nomura Security Financial Research Center)

[2] Some parts of this chapter are quoted from Ohta, Yamagishi, and Saitou (1997).

Figure 2 is based on data until 1990. If recent data are included, the figure looks different, but it just reflects the recent staggering of the equity market and rise of the bond market because of low interest rates in Japan. The normal long-term pattern may be like that shown in Figure 3[3].

Figure 3 Volatility and Expected Return of Assets



Source: Leibowitz (1992)

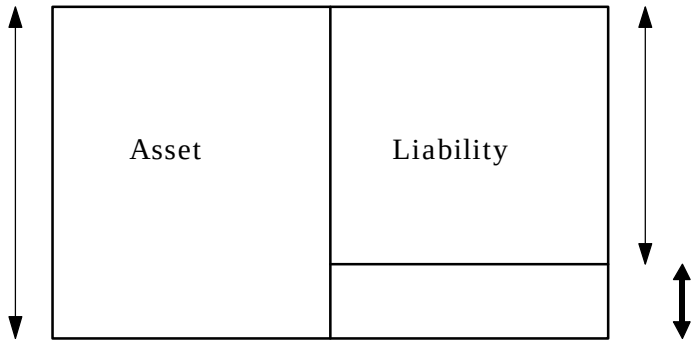
Thus, investments in bonds might be questionable. To clarify the meaning of bond investments Leibowitz (1992) gave the following explanation.

(2) **Explanation of Leibowitz**

Investments in bonds are mainly done by institutional investors, and they usually carry liabilities that have similar cash flow to bonds. The variation of the present value of liabilities is very similar to that of bonds. The risk of investments should never only be considered from the asset side. Investors should compare assets with liabilities to consider the risk of investment. The difference in the change of present value between assets and liabilities is the true risk for institutional investors (see Figure 4).

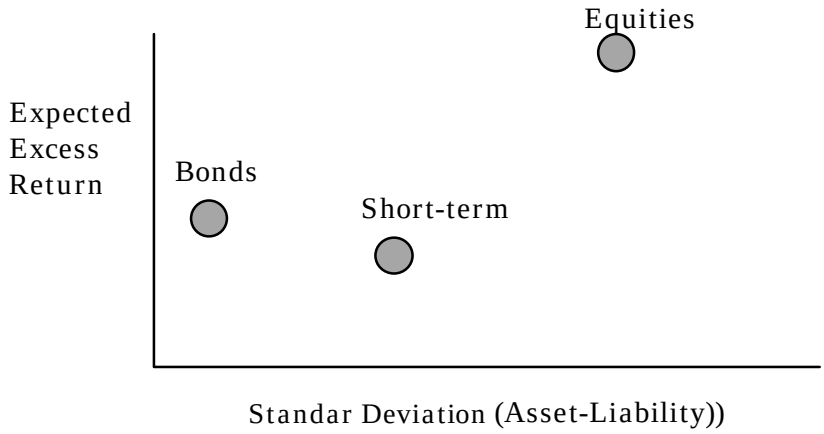
[3] According to CCAPM (Consumption Based Capital Asset Pricing Model) the expected excess return of an asset which has a high correlation to the aggregate consumption should be high. In general, return of equities may have relatively high correlation to the aggregate consumption. But, for fixed income securities, if the consumption is lower because of recession, the interest rate might go down and prices of fixed income securities might go up. So the correlation between returns of fixed income securities and the aggregate consumption may be relatively low. Because of this, the expected return of bonds may be lower than that of equities even if we compare them as risk-adjusted return bases.

Figure 4 Asset, Liability and Risk



The variation of bond prices is very similar to that of liability value. So, if we consider the difference in the change of present value between assets and liabilities as a risk, bonds are low risk assets. Therefore, the relation between risk and return should be like that shown in Figure 5. The price change risk of bond investment is meaningless unless compared with that of liabilities. Generally speaking, institutional investors that carry long-term fixed rate liabilities may be able to reduce the risk of the whole balance sheet by investing in long-term bonds. Institutional investors may effectively reduce the risk to hold bonds of similar maturity to their liabilities.

Figure 5 Difference of Asset and Liability



Source: Leibowitz (1992)

As this explanation by Leibowitz is easy to understand for investors, it has been generally accepted.

If the argument mentioned above is correct, each investor's optimal bond portfolio should

be different because their liability structure is different. In other words, there can not be a common optimal bond portfolio for all investors. However, how can we explain the relationship between this conclusion and the common market portfolio that comes from mean-variance analysis?

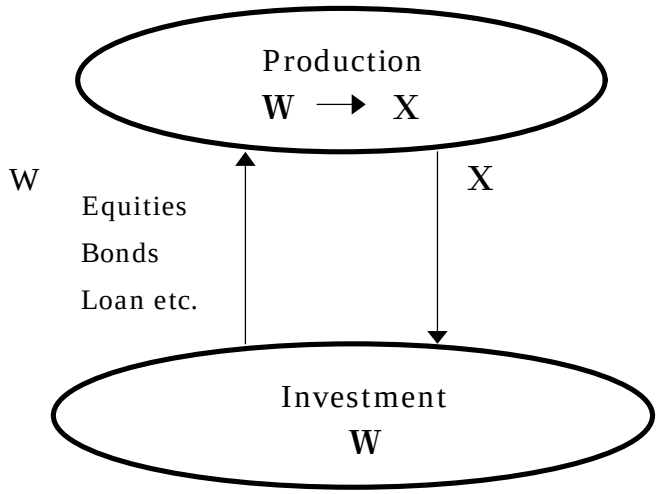
Granito (1987) gave an answer to this problem.

(3) The Security Market

Before explaining Granito's opinion, it may be useful to know his definition of a security market.

Suppose that there are two functions, investment and production (Figure 6), in the market place. Investors begin to invest their wealth (W) in a variety of risk assets issued by producers at first. Producers use them to produce goods. Investors will receive the payoffs in accordance with the conditions described at the time the risk assets were issued. The payoffs consist of dividends, interests, or capital gains (losses) based on the results of production.

Figure **6** Security Market



Source: Nomura Security Financial Research Center

The decision making in investment is just to solve the selection problem of inputs for the production process. If risk assets based on mean-variance analysis are such ones, the suggested market portfolio is an aggregation of securities issued for the purpose of the production of

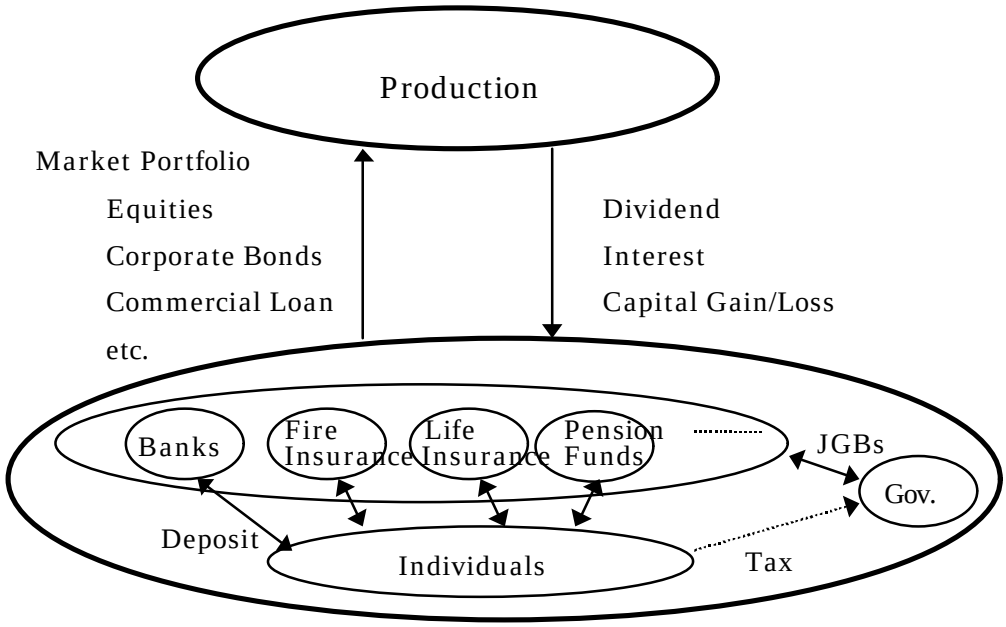
goods.

Most equities in the market are issued for that purpose. But, there are many bond issues the purpose of which is not to obtain funds for production, according to Granito. He argues such bonds should be excluded from the market portfolio.

(4) Explanation of Granito

1) Suppose that an investor, e.g. a pension fund, holds bonds issued by a financial institution. But, both the pension fund and the financial institution are providers of funds for production (Figure 7). In this case, the assets of the pension fund become the liabilities of the financial institution. Is it appropriate to include bonds that are offset among investors in the market portfolio?

Figure 7 Security Market



Source: Nomura Security Financial Research Center

2) Suppose that an investor, e.g. a life insurance company, holds government bonds, which are guaranteed by tax. If we combine the government and the life insurance company, this only means the life insurance company is holding government liabilities as an asset. At the same

time, tax is a liability of individuals and an asset of the government. Therefore, if the life insurance company, government, and the individuals are combined, those liabilities and assets are offset. In such a case, should government bonds be included in a market portfolio?

As a result, bonds issued by financial institutions and the government are excluded from the market portfolio. According to Granito, bonds issued by non-financial corporations and MBS (mortgage backed securities) are included in the market portfolio. With this argument, Granito argues the necessity of much investment in corporate bonds. However, we don't totally agree with his argument, especially in the case of Japan. He argues that corporate bonds are good assets for investment because of their call feature. But most corporate bonds in Japan have no call provision. We think it is not appropriate to give preferential treatment to corporate bonds in Japan.

But we have to accept his argument that most bonds in the real market are not risk assets that are in mean-variance analysis. A portfolio that includes all existing bonds is quite different from the 'market portfolio'. Because a portfolio that holds all existing bonds is not the market portfolio, it is wrong to say the market-capitalized bond portfolio is optimal for every investor. The market-capitalized bond portfolio is not the market portfolio.

With regard to domestic securities, the market portfolio is a domestic equity portfolio mostly. If the market portfolio for equities could be TOPIX, then the market portfolio for domestic bonds is also TOPIX. For example, in Figure 2 the correlation coefficient of the return of long-term bonds and that of TOPIX during the sample period was 0.38. Based on this and using the volatility of long-term bonds during this period (7.3%) and that of TOPIX (28.2%) to calculate the β of the long-term bond against TOPIX, we have

$$\beta = \frac{\rho_{\text{TOPIX}\cdot\text{Long TermBonds}} \cdot \sigma_{\text{Long TermBonds}}}{\sigma_{\text{TOPIX}}}$$

$$= \frac{0.38 \times 0.073}{0.282}$$

$$\approx 0.1.$$

Based on the excess return of TOPIX (about 12%) against the short-term interest rate during this period, the expected excess return of long-term bonds, calculated by CAPM, is

$$\begin{aligned}&\text{Expected Return of Long Term Bonds} - r\\ &= \beta(\text{Return of TOPIX} - r)\\ &\approx 0.1 \times 12\%\\ &= 1.2\% .\end{aligned}$$

This result is roughly consistent with that shown in Figure 2. The prices of bonds are, to this extent, consistent with CAPM, where TOPIX is the market portfolio. In other words, to this extent, the 'market portfolio' of bonds is also TOPIX; i.e. a market-capitalized equity index. Of course this never means the benchmark for bond investment should be TOPIX.

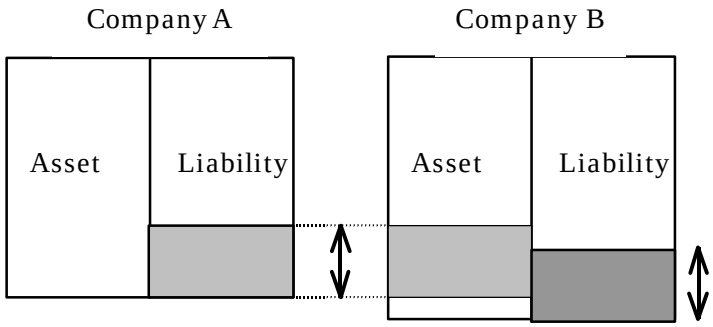
For domestic bonds, the two questions of what is the market portfolio and what should be the benchmark index are quite different.

3. Domestic Equities

Regarding domestic equities, in most cases TOPIX is used as the 'market portfolio' in Japan. However, TOPIX is based on equities listed on the Tokyo Stock Exchange 1st Section. Those listed on the 2nd Section or listed on only local exchanges or the OTC market are not included in TOPIX.

In addition, one of the strange characteristics of the equity market in Japan is the existence of cross-shareholdings among companies. If the market portfolio represents funds invested for production, as shown in Figure 7, because of cross-shareholdings among companies engaged in production, investors would not own a portion of the return, and hence such cross-shareholdings may have to be excluded from the market portfolio. Figure 8 illustrates this problem. In an extreme case, suppose all shares of equities of Company A are held by Company B. The equity price of Company A reflects the activities of Company A. And, through this change in the equity price, the equity price of Company B reflects not only the activities of itself, but also those of Company A.

Figure 8 Effect of Cross-Shareholdings



In this case the market-capitalized index of Company A and B reflect the activities of Company A two times. In other words Company A is double counted. So the equities of Company B should only be included in the 'market portfolio' and Company A should be excluded. Although this is an extreme case, it may be desirable to exclude cross-shareholdings in the 'market portfolio' of equities[4].

In addition to the problems mentioned above, there are unclear points about equities in relation to foreign currency-denominated assets.

4. Foreign Equities

If we introduce securities denominated in foreign currencies (foreign equities and foreign bonds), first of all, we have to examine models of mean variance-analysis, or CAPM extended to cover foreign currencies. Such models were presented by Merton (1973), Solnik (1974), Black (1990), and Adler and Prasad (1992).

(1) Universal Hedge Ratio

Among the above, Black's universal hedge ratio (UHR) model is famous (Black (1989)). This model assumes that

- there are a number of currencies,

[4] One of the examples of indices that are adjusted for this problem is the RUSSEL/NOMURA Japanese Equity Index.

- investors in every country have a utility function, and it is a function of the consumption at next time: i.e. single period model,
- the joint distribution of exchange rates and prices of risk assets in all currencies is known, and returns are normally distributed, and
- there is no trading cost or any restriction for investment.

Then, under general equilibrium,

- investors in every country will hold a common world market portfolio,
- investors in every county will hedge a portion of their foreign-currency denominated assets by a one-period foreign exchange forward contract, and
- the hedge ratios should be universal across all currencies for all investors in any country. The hedge ratio is as follows:

$$1-l=\frac{m_m-s_m^2}{m_m-\frac{1}{2}s_e^2}. \qquad \text{.....(1)}$$

Where m_m = average expected excess return of the world market portfolio, s_m^2 = average variance of excess return of the world market portfolio, and s_e^2 = average variance of exchange rates. Where 'average' means values viewed from each country, weighted by investment amount from each country, and averaged over the countries.

Since investors in every country would have the same hedge ratio for every foreign currency, it is called the universal hedge ratio. It is shown that the hedge ratio is smaller than one, so a fixed portion of foreign currency-denominated assets is not hedged.

From this point two problems arise in the practical asset management. First is that since we have to treat foreign currency-denominated assets and foreign currencies separately, there will be a problem of currency overlay. In other words, how should we treat the hedging of assets denominated in foreign currencies? Another problem is, because the market portfolio for every investor is the common world market portfolio, whether it is right to favor one's own currency denominated assets in asset allocation. In other words, the problem of home bias (i.e. investors

usually over-weight their own country's equities).

(2) Currency Overlay

The reason the currency hedge ratio came to be the same for every investor according to Black's model is probably because of the following assumptions:

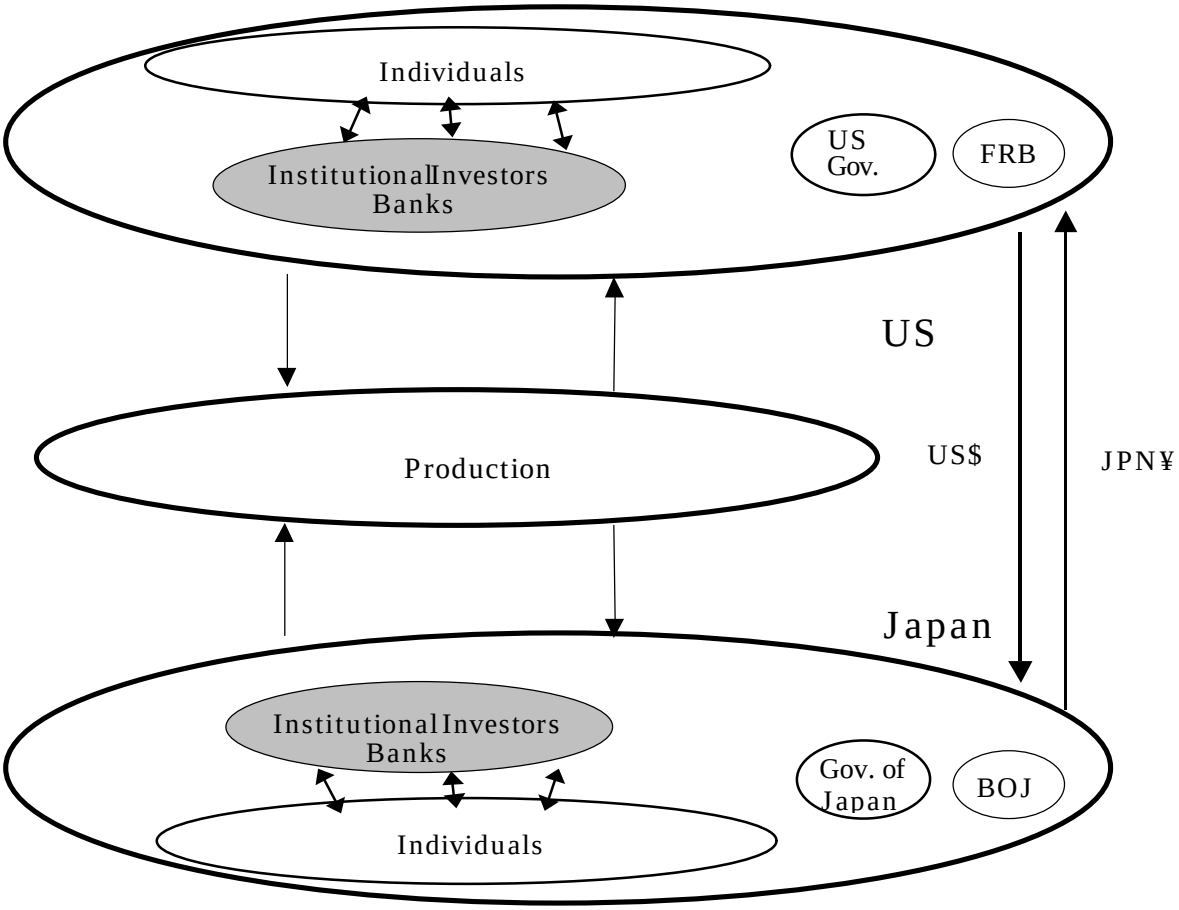
- the net supply of risk-free assets for any currency is zero, and
- the joint distribution of risk asset prices and exchange rates is known (investors agree on this point).

First of all, since the net supply of risk-free assets is zero, when investors in Country A hold a portfolio of risk assets and risk-free assets, investors outside country A should be short of risk-free assets of Country A. So investors in Country B are long in their own risk-free assets and short in the risk-free assets of Country A (i.e. Currency A's discount bonds of one period). This means they are selling Currency A and buying their own at the next time period – a forward foreign exchange hedge of one period. So they are hedging their foreign currency-denominated assets. Since investors know the prices of assets and their distribution, the behavior of all investors is similar and their hedge ratio will be equal. Finally, since investors' risk tolerance is not zero, the hedge ratio is less than one (probably because of their position in risk assets and own risk-free assets).

The main reason the foreign currency hedge ratio is as shown in equation (1) is the net supply of risk-free assets in every country is zero due to macroeconomic constraints. But each investor never faces such constrains. The final adjustment for these constraints is done by an increase (or decrease) in the official reserves of each country. The Foreign Exchange Fund Special Account managed by the Ministry of Finance and the Bank of Japan is included in 'investment'. And the official reserves of Japan are controlled through this account from the viewpoint of the constraints. But each investor is never affected by such constraints (see Figure

9).

Figure 9 International Investment and Security Market



Source: Nomura Security Financial Research Center

Japan should borrow US dollar-denominated short-term funds. The US, on the other hand, should borrow yen-denominated short-term funds. Because the Bank of Japan (BOJ) is included in Japan, the US is hardly a net lender of short-term yen-denominated funds. Japan will be long the risk-free yen assets and short the risk-free dollar assets; Japan is thus hedging the US dollar to that extent.

If we assume some similarity in the utility functions of investors in Japan and the US, it is almost trivial that the hedge ratio will reach equilibrium at a common level for each country. However, for investors who are not affected by macroeconomic constraints, the hedge ratio in equation (1) may not be optimal.

Investors can get foreign currency by ① short selling the foreign risk-free assets, or ②

just trading spot foreign exchange transactions to buy foreign assets and leaving adjustment of the balance of payments to the official reserves. For each investor, ① may not always be optimal.

Whichever decision is made, it is the same for the country as a whole. Japan should do certain exchange rate hedging because of budget constraints. But this hedge ratio for the country as a whole is not necessarily optimal for each investor.

On the other hand, according to Black's model, the universal hedge ratio for every currency may be largely due to the condition that investors agree about the distribution of exchange rates. So if one investor's hedge ratio for one currency is different from those for other currencies, he (or she) is just declaring that he (or she) has a different prediction from that of the market. Then to use a common hedge ratio as a base line (i.e. benchmark) may not be so wrong for practical investment management.

The remaining problem is how to determine the hedge ratio for the benchmark. It will be discussed later.

(3) Home Bias

In Black's model, every investor holds the common world market portfolio of risk assets (except foreign risk-free assets). Since these risk assets are mostly consist of equities, investors may use the world equity market-capitalized index as the benchmark. So preferential treatment of an investor's own country equities is, theoretically, incorrect in this context.

It is difficult to argue whether home bias is rational or not. But one way to build a model with a higher holding of an investor's own country equities in equilibrium is to assume some own state variables for each country and a non-zero correlation between such state variables and the prices of domestic equities. Adler and Prasad (1992) showed that if the inflation rate is stochastic in each country and if there are state variables that are not traded in the market, investment weights in risk assets would be

$$\begin{pmatrix}\mathbf{w}\\ \mathbf{x}\end{pmatrix}=\left[q\,\Sigma^{-1}\begin{pmatrix}\mu_a-r\mathbf{1}\\ \mu_f\end{pmatrix}+(1-q)\Sigma^{-1}\begin{pmatrix}\Sigma_{ap}\\ \Sigma_{fp}\end{pmatrix}\right]$$

$$-\left[H_{yp}\Sigma^{-1}\begin{pmatrix}\Sigma_{ap}\\ \Sigma_{fp}\end{pmatrix}+H_{yz}\Sigma^{-1}\begin{pmatrix}\Sigma_{az}\\ \Sigma_{fz}\end{pmatrix}\right]. \quad \text{.....} \quad (2)$$

Where $\mathbf{w}$ = the vector of investment weights in risk assets, $\mathbf{x}$ = the vector of weights of currencies holdings, Σ = variance-covariance matrix of prices of risk assets and instantaneous forward exchange rates, μ_a = expected return vector of risk assets, and μ_f = expected return of forward exchange rate contracts (risk premium). θ, H_{yp} and H_{yz} are the risk tolerances for respective factors[5] (P= inflation deflator, z= state variables). Σ_{ap} and Σ_{fp} are covariance vectors between asset prices and P and between exchange rates and P respectively. Similarly Σ_{az} and Σ_{fz} are covariance vectors between asset prices and z and between exchange rates and z respectively.

In equation (2), the common part for all investors is the first term of the right hand side. The 2nd and 3rd term is the portfolio for hedging domestic inflation, and the 4th term is the portfolio for hedging the state variables. These are weighted by investor's risk tolerances in equation (2).

For each investor in every country, the three-fund separation is obtained. Those are the world market portfolio, the hedge portfolio for inflation of one's own country, and the hedge portfolio for the state variables of one's country.

The candidate for a hedge portfolio for inflation may be a market-capitalized equity

---

[5] In the model of Adler and Prasad (1992), it is formulated to maximize the expected utility function $E\int_t^T U(C/P,s)ds$, where C/P is real base consumption. From the indirect utility function (J) of this expected utility, these are defined as $q=-J_y/YJ_{yy}$, $H_{yp}=J_{yp}/YJ_{yy}$, $H_{yz}=J_{yz}/YJ_{yy}$, where y is real base wealth ($Y=W/P$).

portfolio of one's own country, such as TOPIX. In this case, own country inflation may be mainly hedged by both own country equities and unhedged foreign currency. Even if we think so, actual investment in domestic equities by domestic investors may be too large according to analysis of recent inflation. However, if Japanese investors are hedging against a critical situation, as during and just after World War II, we may come to the conclusion that their investments are not so concentrated on domestic equities. But still, questions such as the following remain:

① Is a market-capitalized portfolio of domestic equities, such as TOPIX, optimal for every investor for inflation hedging purposes?

② The world 'market portfolio' is an equity portfolio excluding the portion held for hedging purposes. Therefore, it may not be a world market-capitalized portfolio. Is it right to use a market-capitalized world equity index as the 'market portfolio'?

Regarding ①, for hedging against inflation in Japan, TOPIX might be better than foreign equities (when related foreign exchange risk is fully hedged). But TOPIX is just an index of the Tokyo Stock Exchange. Where does the optimality of TOPIX come from? Is there really meaning for 'listed' or 'opened' in Japan? Suppose that a Japanese company stops to be listed on the Tokyo Stock Exchange and starts to be listed on NYSE, and all other conditions are held the same, is it rational for Japanese investors to lower their weight of investment in that company's equity?

Regarding ②, it should be noted that this problem cannot be solved by using an index which consists of the holdings of foreign shareholders. Domestic investors hold a part of their own country's equities for inflation hedging, the rest is held as the domestic portion of the world 'market portfolio'. An equity portfolio held by foreign investors does not include the latter.

In practice, it is very hard to distinguish equity issues according to the purpose of holding. As a result, to specify the world 'market portfolio' is almost impossible.

Even if we may identify such one, it is hard to decide whether it is an optimal portfolio in

equilibrium or a transitional incorrect portfolio caused by investor's misjudgement (e.g. home bias)[6]. So the optimality of a so-called 'market portfolio' may be suspicious in the real world.

5. The Market Portfolio and Optimality

There is someone for whom it is certainly optimal to hold a world market-capitalized portfolio of equities (although there may be the problem of cross-shareholdings). He (or she) is the representative agent.

The representative agent is the hypothetical aggregated agent of all investors. Since all equities in the world have to be held by some investors, the representative agent holds an equity portfolio of total market balances of the world. Since the net supply of risk-free assets is zero, the representative agent holds no risk-free assets and no bonds (except some corporate bonds). And, the representative agent does not hedge currency risks because the agent never holds or short sells the risk-free assets.

Holding a market-capitalized equity portfolio should be optimal for the representative agent. Otherwise, the optimal allocation of funds through the security market would never be realized. So we should give up capitalism and search for a new system.

For the representative agent, investment in the market-capitalized portfolio should be optimal. It is a key doctrine of capitalism to believe that the allocation of funds through the market, that reflects the decisions of all investors, will be optimal (so the representative agent will hold the optimal portfolio). However, the optimal portfolio for the representative agent is not necessarily optimal for each investor. It is hard to believe that the world market-capitalized portfolio composed mostly of equities with no foreign currency hedging is the common optimal portfolio for every investor. At least, it is not appropriate to use this as an optimal benchmark for every investor in a practical sense.

[6] It is sometimes pointed out that home bias may be caused by differences in the information and transaction costs when securities are listed in the home country or a foreign country.

6. Foreign Bonds

Foreign bonds, except for some corporate bonds, may not be included in the world 'market portfolio'. However, there are many investors, e.g. Japanese pension funds, who hold foreign bonds. The reasoning from the models of Adler and Prasad (1992) might lead us to the conclusion that foreign bonds should be effective for hedging inflation or some state variables. It is natural to hold foreign currency-denominated assets to hedge inflation. But we may invest in foreign equities rather than foreign bonds. So this should mean that foreign interest rates rather than foreign exchange rates are effective for hedging.

Even if holding foreign bonds would be effective for hedging, it is hard to show theoretically or practically that a market-capitalized portfolio of bonds of the world is optimal for this purpose.

On the other hand, according to Black (1989), if we add foreign bonds to our portfolio then full hedging for the currency risks of all foreign bonds is optimal. This is because, in Black's model, investors only hold the world equity market portfolio, and if one tries to invest in bonds of another country (say Country B), he (or she) has to short sell B's risk-free assets to get Currency B. Since to short sell B's risk-free assets means hedging Currency B, he (or she) is hedging the foreign exchange risk of Currency B to the extent of the amount of Currency B denominated bonds. But the optimality for the country, which is affected by budget constraints, may be different from that for each investor.

Just like in the case of the universal hedge ratio, investors who bought foreign bonds can either ① hedge the foreign exchange risk by themselves, or ② just make a currency transaction to buy foreign bonds, leave the asset without hedging the currency risk and, as a result, let the official reserves make the necessary adjustment in the balance of payments. For a country as a whole, both are the same, and for the representative agent of Japan, both may be optimal. But it may not always be optimal for each investor to hedge by himself.

Also, as in the case of short-term funds (see Figure 9), because the Japanese government, the largest issuer of yen-denominated bonds, is included in Japan, and the US government, the

largest issuer of dollar-denominated bonds, is included in the US, Japan as a whole may be long in dollar-denominated bonds and short in yen-denominated bonds.

In other words, the representative agent of Japan is selecting a short position in yen-denominated bond as the optimal solution. But does this mean for every investor that it is optimal to hold net yen-denominated bonds short?

Confusion of some Japanese investors about the 'market portfolio' or the universal hedge ratio may come from confusing the optimality for the representative agent and that for each investor.

7. Suggestions for Investments

If we accept the home bias as rational and according to the Adler and Prasad model, a market-capitalized portfolio is not optimal in any sense. This is the case for domestic bonds, foreign bonds, domestic equities, and foreign equities. The only exception is that the world market-capitalized portfolio of equities should be optimal for the representative agent. But this does not mean it is optimal for every investor. The market-capitalized equity portfolio may not be a common 'market portfolio' in the sense of Adler and Prasad (i.e. risk assets excluding the hedge portfolio). Furthermore, there is a possibility that an optimal portfolio for each investor does not exist among the combinations of market-capitalized portfolios of asset classes.

They may not be optimal, but in equation (2) market-capitalized portfolios of each asset class may be as follows:

① foreign equity portfolio is a common risk asset portfolio (1^{st} term),

② foreign equities and foreign bonds without hedging the foreign currency risks and domestic equities are some kind of hedge portfolios (2^{nd} to 4^{th} term).

Domestic bonds may be hedge portfolios for liabilities, and may not be shown in equation (2). But if we set a nominal minimum requirement of consumption in formulating or allow the issue of bonds like liabilities, it is natural to have a hedge portfolio for them as the 5^{th} term.

This may be the domestic bond portfolio. If we set foreign currency liabilities, the foreign bonds will be the hedge portfolio for them.

With these observations, suggestions for investment are as follows:

① In deciding asset allocation, Japanese pension funds sometimes only look at the asset side, and, based on historical variance-covariance and expected return, use QP to get an 'optimal' allocation. However this is very strange.

Domestic equities or domestic bonds should be some kind of hedge portfolio. If we use a formula that does not include what to hedge and try to find the optimal portfolio, a world equity market portfolio should be selected. If this is not the case and a portfolio with larger investment in domestic equities and domestic bonds is selected, it is likely that too high expected returns, compared to the perspective of the market, were assumed. Especially with domestic bonds, higher returns, which are inconsistent with CAPM, might be assumed. If a reasonable expected return is assumed, it is unlikely that domestic bonds would be selected as an optimal portfolio in this way.

Asset allocation should be decided not only from the viewpoint of assets but also that of liabilities.

② Even if liabilities are taken into account, it is unlikely that a meaningful solution can be obtained from using an optimization programming model for the investment weight of domestic equities and hedge ratio of foreign currencies. This part is for hedging inflation. However, an optimal hedge ratio may not be found from the tendency of the recent several years. This hedge is for the situation where Japan alone suffers extraordinary inflation, but such a case is historically very seldom.

For this part, analysis without historical data may be necessary, and it is not appropriate to use the simple optimization programming model to solve the problem. This is not a problem of applied mathematics but may be a problem of art.

③ Compared to the above, problems with domestic bonds are easier to handle by mathematics.

After a temporary decision regarding asset allocation, namely, foreign assets, domestic equities and bonds, examining the composition of bonds, a more desirable bond portfolio for hedging liabilities may be obtained. It may be reasonable to use such a portfolio as the benchmark.

For domestic bonds, it is desirable that a customized index for each investor, reflecting its liabilities, be used as the benchmark. Unlike equities, a market-capitalized bond portfolio is never optimal. Each investor must decide what kind of bonds in the market to invest in. At US pension funds, it is popular to use different sub-indices of bonds as a benchmark for each fund manager for this purpose.

④ Since the market-capitalized portfolio of domestic equities is not optimal for each investor, there might be possibility to use customized portfolios for domestic equities.

However, it is hard to specify a more desirable portfolio for hedging inflation than TOPIX. There is a possibility to use a customized index, but we don't know how to construct it.

Some investors, e.g. believers in some religion, may use a portfolio that excludes equities of particular industries, as a benchmark. For example, excluding the pork industry. Probably, the investor's deflator doesn't include pork, thus the pork industry is not necessary for hedging inflation. Thus, to use such a customized index might be rational.

Even for foreign equities, the market-capitalized portfolio may be different from the 'market portfolio' in the sense of Adler and Prasad and may not be optimal for each investor.

⑤ As for foreign bonds, we know little but for the fact that Japan may be in a long position as a whole. The optimality of a market-capitalized foreign bond portfolio would never be shown. But, investment in a market-capitalized foreign bond portfolio means to purchase more bonds of deficit countries. This might make investors feel unhappy. So some European and US investors use an index of portfolios of foreign bonds weighted by each country's GDP as their benchmark.

Recently, however, an increase in Japanese bond issuance is expected. So we shall welcome investors who would like to buy bonds according to the increase in market-capitalized

weights. From this point of view, it might be better for Japan not to call into question investing in foreign bonds with market-capitalized weights.

⑥ For foreign assets, the universal hedge ratio of foreign exchange rates (equation (1)) may not be optimal for each investor. If an investor assumes an expected return and makes an optimization using formulation as Black, each hedge ratio for currencies would be either 100% or 0% in most cases[7]. It is usual that if you change expected return (risk premium) slightly, the optimization result will change from 100% to 0%, or vice versa.

Unlike the representative agent, since investors are not affected by budget constraints, the universal hedge ratio will never be the solution of such optimization, and the resulting ratio always be 0% or 100% for each currency. It is suspicious that an optimal hedge ratio can be found by such a simple optimization programming. As explained previously, the portion of foreign assets that are not currency hedged is intended to hedge the inflation of the home country. It is suspicious that it is possible to obtain a good suggestion using historical data. Some kind of art may be necessary in this field.

In practice, however, in order to avoid the instability of the solution obtained from a precise model, an approximate round number may be used in a model. For example, it is sometimes assumed that the expected return of any foreign asset is same over currencies. It is not scientific, but some kind of art. But, in order to avoid an extreme solution, this kind of rule of thumb might be practical.

To restrict the candidates of a benchmark for currency hedging to the class of a constant hedge ratio could be this kind of rule of thumb and might be prudent. But the ratio cannot be determined from either theoretical or historical analysis.

It might also be a practical solution to decide, “For all currencies, hedge 50% and the remaining 50% unhedged,” as the benchmark, like Gestineau (1995),

[7] We assume that there is a restriction that the hedge ratio should be between 0% and 100%. An example of the literature on this kind of analysis is ”An Algorithm for International Portfolio Selection and Optimal Currency Hedging” by Rudolf, M. and Zimmermann, H., which appears in Ziemba, Mulvey (1998).

8. Conclusions

Regarding international investment, the 'world market portfolio' in Black's sense is a market-capitalized portfolio which mainly consists of equities without currency hedge. For the representative agent, this should be optimal. But this does not mean that it is optimal for each investor, who is a part of the representative agent, to invest in just only this market portfolio. It is not enough for investors to invest in only such a portfolio.

On the other hand, the market-capitalized portfolio of domestic equities, domestic bonds, and foreign bonds, which some investors in Japan were falsely thinking to be 'market portfolios', are not market portfolios and may not be optimal.

There is a possibility for each investor to customize indices of domestic bonds, domestic equities, foreign bonds, and foreign equities and use them as benchmark indices. Especially, for domestic bonds, as the objectives of hedging are far clearer than other assets, it is a real problem. Institutional investors' holdings of bonds should reflect maturity and necessary liquidity requirements from their liabilities. In contrast to domestic bonds, it is hard to decide how to customize the benchmark for domestic equities, foreign bonds, or foreign equities. In practice, we know little about international investment yet.

The use of a market-capitalized portfolio as a benchmark, although its optimality is suspicious, is not so meaningless in practice. A market-capitalized index shows the average performance of all investors. To compare performance with the average is not so meaningless. However, this index is not optimal for each investor. So the deviation of performance from the index is not a risk for investor. For example, a fund manager, asked by a sponsor to use TOPIX as his (or her) benchmark, should try to outperform TOPIX and be care about TOPIX to this extent. But it is not good to try to trace TOPIX exactly. It does not mean a reduction in risks for the sponsor to reduce deviation from TOPIX on a daily or monthly basis. It will not be a fruitful

effort[8].

Some investors in Japan want to reduce tracking error extremely. But, since a larger deviation from the index does not mean more risks for the sponsor, ranking by excess return of funds divided by deviation from index (such as standard deviation of the tracking error) is meaningless.

Of course, in practice, it is necessary to restrict assets that can be invested in or deviation from the benchmark. Otherwise, asset allocation of the sponsor becomes unclear. However, if it is managed within the permitted guidelines, reducing deviation from the benchmark is mostly meaningless. Since a market-capitalized portfolio shows only average performance, to be concerned about deviation is as if you are scolding your son because he got a different result (either way) from the average of his class.

As for domestic bonds, due to the deficit of the Japanese government, an increasing issuance of Japanese government bonds (JGBs) is expected and 5-year and 30-year JGBs were introduced recently. In addition, due to reform of the Fiscal Investment and Loan Program (FILP) of Japanese government, FILP agency bonds and FILP bonds will be introduced. Moreover, with the progress of so-called Japanese 'Big Bang', further expansion of the corporate bond market is expected. With such progress, characteristics of the Japanese market-capitalized bond index, such as NOMURA-BPI, are expected to change. Anyway, usage of a customized index or sub-indices of Japanese bonds as a benchmark will become necessary in the near future.

< References >

Adler, M. and Prasad, B. (1992) "On Universal Currency Hedges." *Journal of Financial and Quantitative Analysis* (March).

[8] It is shown that the solution from minimizing the variance of the difference of returns between the portfolio and the benchmark, given the condition that the expected excess return from the benchmark is constant, is usually not an efficient portfolio (see Roll (1992)). Because of this, it is also not recommended to minimize the tracking error from the benchmark.

Black, F. (1989) “Universal hedging: Optimizing currency risk and reward in international equity portfolios.” *Financial Analysts Journal* (July-August).

Black, F. (1990) “Equilibrium Exchange Rate Hedging.” *Journal of Finance* (July).

Gastineau, G. (1995) “The Currency Hedging Decision: A Search for Synthesis in Asset Allocation.” *Financial Analysts Journal* (May-June).

Granito, M. (1987) “The Problem with Bond Index Funds.” *Journal of Portfolio Management* (Fall).

Huang, C. and Litzenberger, R. (1988) *Foundations for Financial Economics*, North-Holland.

Ingersoll, J. (1987) *Theory of Financial Decision Making*, Rowman & Littlefield.

Leibowitz, M. (1992) *Investing*, Probus.

Markowitz, H. (1959) *Portfolio Selection: Efficient Diversification of investment*, Wiley.

Merton, R. (1973) “An International Capital Asset Pricing Model.” *Econometrica* (September).

Merton, R. (1990) *Continuous-Time Finance*, Basil Blackwell.

Ohta, T. (1996) ”Problems of Bond Index Investment.” *The Nikkei Financial Daily* (October) (written in Japanese).

Ohta, T., Yamagishi, Y. and Saitou, I. (1997) ”Development of Bond Semi-Customized Index.” *Zaikai-kansoku*, Nomura Research Institute (written in Japanese).

Roll, R. (1992) “A Mean/Variance Analysis of Tracking Error.” *The Journal of Portfolio Management* (Summer).

Solnik, B. (1974) “An Equilibrium Model of the International Capital Market.” *Journal of Economic Theory* (August).

Ziemba, W. and Mulvey, J. (Editor) (1998) *Worldwide Asset and Liability Modeling*, Cambridge University Press.