This article was translated by the author and reprinted from the August 2011 issue of the Securities Analysts Journal® with the permission of the Securities Analysts Association of Japan(SAAJ).

Volatile realized idiosyncratic volatility

Takehide Hirose, CMA
Yasuhiro Iwanaga

Table of contents

This article investigates the reason why stocks with higher long-term realized idiosyncratic volatility tend to have lower future returns in Japan. We find that both time-varying realized volatility and long-term return reversal explain this anomaly. Our results suggest that realized idiosyncratic volatility is not appropriate proxy for expected idiosyncratic risk.

Takehide Hirose

Equity Quants Team Leader, Indexing and Quantitative Investment Department, Sumitomo Trust and Banking Co., Ltd.
Graduated from the School of Engineering, the University of Tokyo (M.En), in 1996. After working in Plaza Asset Management and Mitsui Trust and Banking, he joined Sumitomo Trust and Banking in November 2007. He is at the current post since April 2008. He has completed his master's degree of Business Administration at the School of Business Sciences, the University of Tsukuba, in 2004 and his doctoral degree at the School of Business Administration, Kobe University in 2007. His works include "Can Margin Traders Predict Future Stock Returns in Japan?" (Pacific-Basic Finance Journal (co-author), 2006 Asia FA/FMA Conference best paper).

Yasuhiro Iwanaga

Quants analyst, Indexing and Quantitative Investment Department, Sumitomo Trust and Banking Co., Ltd.
Graduated from the School of Economics, Osaka University and joined Sumitomo Trust and Banking in 2002. After working in asset management department, he has been at his current post since April 2007. Earned a MBA in Finance from Waseda University in 2007.

1. Introduction

Prices are discounted to compensate for the future uncertainty. This concept leads an idea that investments with higher uncertainty have higher future returns. This is a well-known basic concept (theory) in investment. This explains why stocks have higher returns than bonds in long-term investment.

On the other hand, the theoretical framework such as Merton [1987] predicts that volatility of the residual returns that cannot be explained by risk factors ("idiosyncratic volatility") has a positive correlation with future returns because investors who cannot fully diversify their portfolios due to market imperfections will require a risk premium.

There are many empirical studies devoted to the investigation of the cross-sectional relationship between volatility and future returns, but some are not consistent with these theories. Ang *et al.* [2009] find that in the equity markets of developed countries, stocks with higher idiosyncratic volatility tend to have lower future returns, and document that the phenomenon cannot be explained by transaction costs, institutional investor ownership ratios or return skewness. Yamada and Nagawatari [2010], in a recent analysis of Japanese equity markets, document that there is a negative relation between total volatility and future returns and that this relation is caused by excessive expectations for high-volatility stocks by investors and security analysts and expectations of accidental huge returns recorded among high-volatility stocks.

Some papers find counter evidence against a negative relationship between idiosyncratic volatility and future returns. Bali and Cakici [2008] examine the robustness of the relationship between idiosyncratic volatility and future returns under the various conditions and show that data frequency used to estimate idiosyncratic volatility and weighting method used to compute average portfolio returns play a critical role in determining the statistical significance of the negative relationship between idiosyncratic volatility and future returns. Fu [2009] and Huang *et al.* [2010] find a negative relationship between realized idiosyncratic volatility and future returns when historical data is used in the calculations, but a positive relationship when expected idiosyncratic volatility is estimated using the EGARCH model. They also indicate that the negative relationship between realized idiosyncratic volatility and future returns can be explained by the short-term reversal effect.

Most of the prior empirical papers in other countries out of Japan analyze idiosyncratic volatility observed over the short period of one month, while the Japanese prior researches analyze total volatility calculated from long-term monthly returns such as 60 months. In addition, while some papers in other countries provide powerful counter evidence against the negative relationship between idiosyncratic volatility and future returns for the short-term, as far as the authors know, there have been no papers written with the intent to disprove the negative relationship between total volatility and future returns over the long periods of time observed in Japanese equity markets.[1]

This paper analyzes long-term volatility in Japanese equity markets and finds that the relationship between long-term volatility and returns may not be a puzzle that is inconsistent with theory. In this paper, the term "total volatility effect" is used to refer to the phenomenon in which stocks with higher volatility of total returns tend to have lower future returns; "idiosyncratic volatility effect" to refer to the phenomenon in which stocks with higher idiosyncratic volatility tend to have lower future returns. Focusing on the idiosyncratic volatility effect, which is presumed to be the main factor in the total volatility effect, this paper explores the question of whether the idiosyncratic volatility effect is observed because realized idiosyncratic volatility is used as a proxy for expected idiosyncratic volatility.

In practice, volatility computed from prior returns is usually used as an alternative to the estimated volatility in the subsequent period. Practitioners use an implicit assumption that future risk structures will not differ very much from the structures estimated by the past returns. This paper demonstrates that realized idiosyncratic volatility measured with prior returns is low-sustainability and moves like mean-reverting. If it is assumed that realized idiosyncratic volatility moving in this manner is a proxy for expected idiosyncratic volatility, then a portfolio with high idiosyncratic volatility will include numerous stocks of which future idiosyncratic volatility is lower than that of realized idiosyncratic volatility, which explains the idiosyncratic volatility effect. The mean-reverting movement observed for realized idiosyncratic volatility is related to prior returns, and part of the idiosyncratic volatility effect measured with realized idiosyncratic volatility can be explained by the impact of the long-term reversal effect. This paper also reports that the idiosyncratic volatility effect is not a statistically significant effect once the long-term reversal effect is eliminated.

The remainder of this paper is organized as follows. The next chapter explains the volatility used in this paper. Chapter 3 demonstrates that the idiosyncratic volatility effect is the primary factor in the total volatility effect. Chapter 4

[1] Cao and Xu [2010] analyzes US equity markets by breaking down the long-term and short-term components of idiosyncratic volatility. They find a positive relation between long-term components and future returns. However, its purpose is to identify the reason why the results of analyses using short-term realized idiosyncratic volatility are different from the results of analyses using expected idiosyncratic volatility estimated from EGARCH or similar forecasting models. Its intent is not to disprove the long-term idiosyncratic volatility effect.

demonstrates that the use of realized idiosyncratic volatility calculated on the basis of prior returns is one factor in the idiosyncratic volatility effect. Chapter 5 contains additional consideration regarding the idiosyncratic volatility effect. Chapter 6 contains the paper's conclusions.

2. Data and Methodology

This paper analyzes monthly data for stocks listed on the First Section of the Tokyo Stock Exchange. The period covered in this study is January 1980 to January 2011. We use two type of dataset, market data (stock prices, returns etc.) is from QUICK-Astra and financial data is from Nikkei NEEDS.

In this chapter, definitions of several type of volatilities used in this paper are explained. Total volatility (henceforth "TVOL") is defined as realized volatility for the most recent 60 months (minimum of 36 months), and is calculated with the following formula.[2]

Total volatility:

$$TVOL_{it} = \sqrt{\sum_{t=1}^{T}(r_{it} - \bar{r}_{it})^2 / (T-1)} \ldots (1)$$

In this formula, i represents the individual stock; t, the point in time (monthly); T, the number of points in time; r_{it} the individual stock's monthly excess return relative to the short-term interest rate; and $\bar{r}_{it}$, the average value during the period of r_{it}.

In this paper, the Fama and French [1993] three-factor model ("FF3 Model") is employed to break down TVOL into systematic volatility ("SVOL") and idiosyncratic volatility ("IVOL").

$$r_{it} = \alpha_i + \beta_i MKT_t + \gamma_i SMB_t + \eta_i HML_t + \varepsilon_{it} \ldots (2)$$

where MKT_t is the market portfolio's monthly excess return against the short-term interest rate; SMB_t, the monthly return for the "Small cap Minus Big" (SMB) factor; HML_t, the monthly return for the "High book/price Minus Low" (HML) factor; α_i, the intercept; ε_{it}, the regression residual.[3]

---

[2] A volatility measurement period of 60 months is used to be consistent with Ishibe *et al.* [2009], Yamada and Uesaki [2009], Yamada and Nagawatari [2010] and other prior research in Japan.

[3] The MKT, SMB and HML return series are calculated using the approach found in Kubota and Takehara [2007] with stocks listed on the First Section of the Tokyo Stock Exchange as the universe.

We compute residuals using the most recent 60 months (minimum 36 months) data for each stock. We identified the standard deviation of the model residuals as IVOL.

Realized idiosyncratic volatility:

$$IVOL_{it} = \sqrt{\sum_{t=1}^{T} \varepsilon_{it}^2 / (T-1)} \ \ldots \ (3)$$

SVOL is calculated with the following formula.
Systematic volatility:

$$SVOL_{it} = \sqrt{TVOL_{it}^2 - IVOL_{it}^2} \ \ldots \ (4)$$

TVOL, IVOL and SVOL are realized volatility calculated from prior data. We also use future idiosyncratic volatility which is calculated from future data ("FVOL"). FVOL is defined as future IVOL for the subsequent 60 months, which is calculated with the following formula.[4]

Future idiosyncratic volatility:

$$FVOL_{it} = IVOL_{i(t+60)} \ \ldots \ (5)$$

Because subsequent 60 months stock returns are used to calculate FVOL, test period is different in case of using FVOL. It ends in January 2006.

3. Idiosyncratic volatility effect

In this chapter, we investigate whether there is a cross-sectional relationship between individual volatility and future returns. We conduct quintile analysis. All stocks are sorted by volatility cross-sectionally normalized within the 33 sectors of the Tokyo Stock Exchange. Normalized value is used in order to eliminate the influence of sector bias. Five equal-weighted portfolios are constructed and rebalanced each month. Sorted portfolios are constructed by descending order. The largest volatility is Q1 portfolio and the smallest volatility is Q5 portfolio.

[4] The FVOL measurement period and the Quintile portfolio analysis return measurement period overlap, but the conclusions of this paper do not change even with a lag of 1 month. Results are presented without the lag for ease of understanding.

Exhibit 1 presents the equal-weighted returns of five portfolios that are formed by sorting stocks based on TVOL, SVOL and IVOL, which are found in Panels A, B and C respectively.

We begin by confirming TVOL (Panel A). The return of a long-short portfolio (Q1-Q5) in which the highest TVOL portfolio (Q1) is long and the lowest TVOL portfolio (Q5) is short is -4.40% (t-value -1.38), and higher TVOL portfolios have lower returns. To the contrary, risk (standard deviation of return when investing in the ranking portfolio) is highest for Q1 (29.23%) and lowest for Q5 (17.62%).[5] We confirm the total volatility effect reported in many papers.

Exhibit 1 Quintile portfolio performance

Panel A: TVOL quintile portfolio				Panel B: SVOL quintile portfolio				Panel C: IVOL quintile portfolio			
	Return	Risk	t-value		Return	Risk	t-value		Return	Risk	t-value
Q1 (High)	3.60	29.23	0.63	Q1 (High)	6.72	30.32	1.13	Q1 (High)	2.48	27.25	0.46
Q2	6.59	25.18	1.33	Q2	8.05	25.30	1.62	Q2	6.66	24.19	1.40
Q3	7.77	22.80	1.74	Q3	7.13	22.39	1.62	Q3	7.16	22.36	1.63
Q4	7.99	20.24	2.01	Q4	6.79	19.86	1.74	Q4	8.21	21.37	1.96
Q5 (Low)	8.00	17.62	2.32	Q5 (Low)	5.30	17.34	1.56	Q5 (Low)	9.44	19.21	2.51
Q1- Q5	-4.40	16.24	-1.38	Q1- Q5	1.42	17.58	0.41	Q1- Q5	-6.96	12.30	-2.88

Note: Figures in Return column represent the average value of returns for the individual quintile portfolio or long-short portfolio (Q1-Q5); risk represents an annualized translation of the standard deviation of return for the individual quintile portfolio or long-short portfolio. t-values are t-statistics against the null hypothesis that the average returns for the individual quintile portfolio or long-short portfolio is zero.
Source: Created by the authors, and so throughout.

Next, our focus moves to SVOL (Panel B). The return of the long-short portfolio is 1.42% (t-value 0.41), which is not statistical significant but is nonetheless positive. Rank-by-rank returns indicate that higher SVOL portfolios tend to have higher returns. When we take a look at the highest SVOL portfolio (Q1), its return tends to be a little lower than that expected from its risk. However, Q1 portfolio's low return is not enough to explain the total volatility effect.

Finally, we confirm IVOL (Panel C). The return of the long-short portfolio is -6.96% (t-value -2.88), which is statistically significant and negative. Rank-by-rank returns indicate that higher IVOL portfolios tend to have lower returns, and extremely low for the highest IVOL portfolio (Q1). Similarly, higher IVOL portfolios provide higher risks.

[5] Covariance is not taken into account in the constructing of the Quintile portfolio, so in relation to the future risk of the portfolio, the results are merely observations rather than expectations.

As can be seen from Exhibit 1, the idiosyncratic volatility effect appears to be the primary factor in the total volatility effect reported in prior research on Japanese equity markets. The remainder of this paper therefore investigates the idiosyncratic volatility effect in detail.

4. Volatile realized idiosyncratic volatility

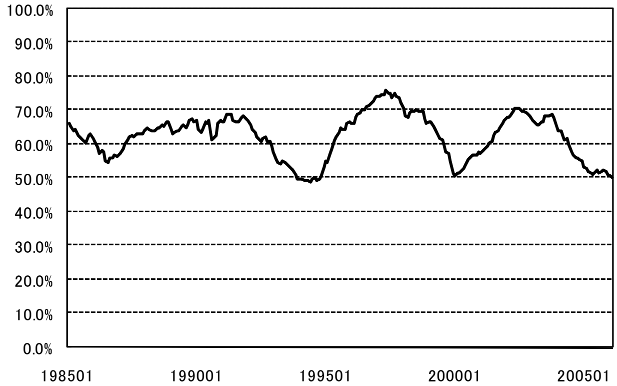
4.1 Sustainability of idiosyncratic volatility

The finding that stocks with higher idiosyncratic volatility tend to have lower future returns is not consistent with modern finance theory. Fu [2009] and Huang *et al.* [2010] indicate that the short-term idiosyncratic volatility effect may potentially be the result of using realized idiosyncratic volatility calculated from prior data as a proxy for expected idiosyncratic volatility. Intuitively as well, it is not rational to believe that future idiosyncratic risk will be high for a stock that has experienced significant news events over the past several years, if news events unique to the stock are purely random events that follow a Poisson process. We therefore begin by verifying the sustainability of idiosyncratic volatility.

In order to confirm the sustainability of idiosyncratic volatility, we compare realized idiosyncratic volatility and future idiosyncratic volatility.[6] We perform a variance ratio test (F test) between IVOL and FVOL for each individual stock at each point in time and calculate the percentage of stocks for which F values are statistically different at the 5% confidence level per total number of eligible stocks in the portfolio at each point of time. This figure means percentage of stocks for which the difference between IVOL and FVOL is significantly large. If IVOL is sustainable, this percentage should be low. The results are shown in Exhibit 2. We find that between 50% and 75% of stocks have statistically significant differences between IVOL and FVOL (average 62.3%). Therefore, realized idiosyncratic volatility measured with prior data appears to be different from the real realized idiosyncratic volatility of the future.

Exhibit 2 Percentage of stocks with statistically significant differences between IVOL and FVOL

[6] This paper focuses on volatility observed over the long period of 60 months. The EGARCH model analysis found in Fu [2009] and Huang *et al*. [2010] was not deemed to be realistic because of problems with the size of the sample. Analysis using a volatility forecasting model is one of the issues identified at the end of the paper.



Note: Expresses the percentage of stocks with statistically significant differences between IVOL and FVOL at a confidence level of 5% in variance ratio tests (F tests) performed at each point in time.

4.2 FVOL effect

We next examine what happens to the relationship between idiosyncratic volatility and future returns if FVOL is used in place of IVOL as expected idiosyncratic volatility.[7] Exhibit 3 shows the performance of quintile portfolios which is constructed by sorting stocks according to IVOL, FVOL and the logarithmic values for $FVOL^2/IVOL^2$ ("VRATE") in Panels A, B and C, respectively.

We begin by confirming Panel A. The difference from Panel C in Exhibit 1 is the analytical period.[8] It can be confirmed that the idiosyncratic volatility effect is a stable effect regardless of the analytical period.

Exhibit 3　Quintile portfolio performance

Panel A: IVOL quintile portfolio				Panel B: FVOL quintile portfolio				Panel C: VRATE quintile portfolio			
	Return	Risk	t-value		Return	Risk	t-value		Return	Risk	t-value
Q1 (High)	4.93	27.47	0.82	Q1 (High)	19.10	28.55	3.07	Q1 (High)	19.90	25.39	3.60
Q2	9.62	24.64	1.79	Q2	11.75	24.35	2.21	Q2	13.88	23.20	2.75
Q3	10.04	23.00	2.00	Q3	9.05	22.55	1.84	Q3	9.95	22.25	2.05
Q4	11.17	21.98	2.33	Q4	6.66	20.90	1.46	Q4	5.86	21.86	1.23
Q5 (Low)	12.71	19.90	2.93	Q5 (Low)	3.85	18.79	0.94	Q5 (Low)	1.34	22.30	0.28
Q1- Q5	-7.78	12.12	-2.95	Q1- Q5	15.25	13.19	5.31	Q1- Q5	18.55	7.38	11.55

Note: See the notes to Exhibit 1.

[7] In actual practice, expected idiosyncratic volatility is estimated from implied volatility or some form of time-series model and differs from FVOL.
[8] Panel A of Exhibit 3 measures performance for the period for which it is possible to measure FVOL.

Next we confirm Panel B. Comparing the Panel A results against the Panel B results for the same ranking portfolio risk (standard deviation of return when investing in the ranking portfolio), no apparent difference is found. Turning to returns, however, there are large differences between IVOL results and FVOL results. Higher FVOL portfolios have higher returns. For FVOL, the return of the long-short portfolio is 15.25% (t-value 5.31), which is statistically significant and positive. Therefore, if an investor could perfectly predict future idiosyncratic volatility, no idiosyncratic volatility effect would be observed. In other words, if future idiosyncratic volatility is correctly estimated, stocks with higher idiosyncratic volatility would have higher future returns, which indicate that, the use of realized idiosyncratic volatility as a proxy for expected idiosyncratic volatility may be one factor in the idiosyncratic volatility effect.

Finally, turning to Panel C, returns are higher in higher VRATE portfolios. For VRATE, the return of the long-short portfolio is 18.55% (t-value 11.55), which is statistically significant and positive. This finding indicates that future returns will tend to be low when realized idiosyncratic volatility is higher than future idiosyncratic volatility.

4.3 Relationship between IVOL and FVOL

Next we examine the relationship between IVOL and FVOL. The first line of Exhibit 4 shows the average VRATE of portfolios sorted on IVOL. We also performed a variance ratio test (F test) for each stock at each point in time with a confidence level of 5%, and the second line (the third line) contains a time series average value of the percentage of stocks for which FVOL is lower (higher) than IVOL at a statistically significant level.

Exhibit 4 indicates that the higher IVOL the portfolio, the lower the average VRATE. Likewise, the higher IVOL the portfolio, the larger the percentage of stocks for which FVOL is lower than IVOL at a statistically significant level. Conversely, the higher IVOL the portfolio, the lower the percentage of stocks for which FVOL is higher than IVOL. The remarkable results are seen in the highest IVOL portfolio. 64.11% of stocks have lower FVOL than IVOL at a statistically significant level and only 10% have higher FVOL. In other words, the higher realized idiosyncratic volatility, the higher possibility that future idiosyncratic volatility will decline. This tendency is most pronounced among stocks belonging to the highest IVOL portfolio.

Exhibit 4 Relationship between IVOL and FVOL in the IVOL quintile portfolio

	Q1 (High)	Q2	Q3	Q4	Q5 (Low)	Q1−Q5
VRATE	−0.67	−0.25	−0.07	0.12	0.39	−1.06
FVOL < IVOL	64.11%	44.69%	34.37%	23.93%	14.73%	49.38%
FVOL > IVOL	9.28%	18.87%	24.63%	31.46%	45.21%	−35.93%

Notes

1: VRATE expresses the average value of VRATE for a quintile portfolio created on the basis of IVOL. (For Q1-Q5, it expresses the difference in VRATE average values for the Q1 portfolio and Q5 portfolio.)

2: FVOL <IVOL (FVOL> IVOL) expresses the time-series average value of the percentage of stocks for which variance ratio tests (F tests) find FVOL to be lower (higher) than IVOL at a statistically significant level with a confidence level of 5%. (For Q1-Q5, it expresses the difference in FVOL <IVOL (FVOL> IVOL) between the Q1 portfolio and Q5 portfolio.)

Results so far indicate that many of the stocks with high (low) realized idiosyncratic volatility measured with prior data will see their idiosyncratic volatility decline (increase) in the future. Those stocks for which idiosyncratic volatility declines (increases) will have extremely low (high) future returns, so stocks with higher (lower) idiosyncratic volatility tend to have lower (higher) future returns.

5. Discussion and Consideration

Our findings indicate that realized idiosyncratic volatility is different from real future idiosyncratic volatility and that the mean-reverting movement of realized idiosyncratic volatility is one factor in the idiosyncratic volatility effect. This chapter provides additional analysis to enhance the interpretation of the idiosyncratic volatility effect.

5.1 Description of phenomenon

Consider the following case in the interpretation of the idiosyncratic volatility effect.

"Company A makes a public announcement that it has successfully captured a large share of its market and earned a large profit. The market is confident that Company A will have further successes in the future, and there is a large rise in the stock price."

How should investors take idiosyncratic risks of this stock? If idiosyncratic volatility measured with prior returns is used, there will be a sharp jump in idiosyncratic risk as a result of this event. This judgment, however, tacitly assumes that realized idiosyncratic volatility will be sustained into the future, and the results of the preceding chapter contradict that.

On the other hand, if the stock specific event is a random occurrence that follows a Poisson process, the idiosyncratic risk expected for the stock will be unrelated to past events. The reason why realized idiosyncratic volatility is observed to move like mean-reverting in Exhibit 4 is because realized idiosyncratic volatility has been changed as a result of past events even though there has been no change in expected idiosyncratic risk, and this movement can be interpreted as a return to expected levels. However, in Panel C of Exhibit 3, the returns during mean-reverting for realized idiosyncratic volatility are too large simply for the reverting of realized idiosyncratic volatility to expected levels, so it may be that there is some form of overreaction at work.

Fu [2009] and Huang *et al.* [2010] indicate that the short-term idiosyncratic volatility effect can be explained as a short-term reversal effect. As is the case with VIX, which is known as a fear index, if future idiosyncratic volatility is high/low when prior returns are low/high, the long-term reversal effect may explain the long-term idiosyncratic volatility effect.[9]

5.2 Realized idiosyncratic volatility and prior returns

This paper uses the FF3 Model intercept ("FFINCP") as a proxy for prior average returns to verify the relationship between realized idiosyncratic volatility and prior average returns.[10] The FFINCP is a metric expressing the average level of abnormal returns during the past 60 months as measured by the FF3 Model.

In Exhibit 5, Panel A shows the relationship between FVOL and IVOL for five portfolios sorted on FFINCP. Each value in the body of the table is calculated by the same way as of Exhibit 4. Panel B contains the performance of FFINCP sorted portfolios.

Initially, we examine the persistence of idiosyncratic volatility. Finding as far is suggested that lower future idiosyncratic volatility relative to realized idiosyncratic volatility is observed among higher FFINCP stocks. We can confirm from Panel A that the average value of VRATE is lower in higher FFINCP portfolios. Higher FFINCP portfolios have larger percentage of stocks

9 Though omitted for reasons of space, the short-term reversal effect is unable to explain the long-term idiosyncratic volatility effect.

10 The FF3 Model intercept was used rather than raw returns because the analysis in this paper focuses on idiosyncratic volatility.

for which FVOL are lower than IVOL at a statistically significant level; conversely, higher FFINCP portfolios have lower percentage of stocks for which FVOL is higher than IVOL at a statistically significant level.

Exhibit 5 Relationship between FFINCP and the realized idiosyncratic volatility/future returns

Panel A: Relationship between IVOL and FVOL in the FFINCP quintile portfolio

	Q1 (High)	Q2	Q3	Q4	Q5 (Low)	Q1-Q5
VRATE	-0.42	-0.14	-0.04	0.05	0.10	-0.52
FVOL < IVOL	52.25%	38.25%	33.25%	29.30%	27.12%	25.13%
FVOL > IVOL	17.41%	23.77%	27.30%	30.17%	32.45%	-15.05%

Panel B: Performance of the FFINCP quintile portfolio

	Q1 (High)	Q2	Q3	Q4	Q5 (Low)	Q1-Q5
Return	-1.18	4.48	7.05	9.98	13.62	-14.80
Risk	21.92	21.28	22.15	23.07	25.88	9.45
t-value	-0.27	1.07	1.62	2.21	2.68	-7.99

Note: For Panel A, see the notes to Exhibit 4. For Panel B, see the notes the Exhibit 1

Next we confirm Panel B. Returns are lower in higher FFINCP portfolios; the long-short portfolio's return is -14.80% (t-value -7.99), which is statistically significant and negative. These results suggest that idiosyncratic volatility effect is a similar to a long-term reversal effect in which future returns are low for stocks with high abnormal returns in the past. Since we have observed that higher FFINCP portfolios have larger number of stocks that will experience lower future volatility than realized volatility estimated from prior returns, it is possible that the idiosyncratic volatility effect measured with realized idiosyncratic volatility contains an effect that can be explained by the long-term reversal effect.

5.3 Relationship with the long-term reversal effect

This section uses a Fama-MacBeth regression analysis to confirm the possibility for the idiosyncratic volatility effect to contain an effect that can be explained by the long-term reversal effect. The Fama-MacBeth regression analysis begins by performing a cross-section regression analysis at each point in time and then computing the time-series average of regression coefficients. Test method is as follows. First, a cross-section regression analysis is performed using the next month's stock returns as the dependent variable, and IVOL, beta, the logarithmic value of market capitalization, the logarithmic value of B/P and a sector dummy as independent variables.[11] This cross-section regression is performed by several types. Some include FFINCP and others do not. We check the change of the explanatory power of regression coefficient against IVOL by the addition of FFINCP to the list of independent variables in the regression. Note that the weight of individual stock in the cross-section regression is proportional to the square root of each stock's market capitalization.[12]

Exhibit 6 contains the average value of the regression coefficient by the Fama-MacBeth regression analysis and the p-value of a two-sided test against the null hypothesis that the average value of the regression coefficient is zero.

We begin by confirming A1. Even adjusting the major variables used in the FF3 Model, the regression coefficient against IVOL is -0.13 (p-value 1.9%) which is negative and statistically significant with a confidence level of 5%. This suggests that the idiosyncratic volatility effect cannot be explained by the FF3 Model.

Exhibit 6 Fama-MacBeth regression analysis results

	A1	A2	A3
IVOL	−0.13		−0.05
	1.9%		41.9%
Beta	0.17	0.12	0.13
	4.0%	19.4%	12.3%
Logarithmic market capitalization	−0.03	0.01	−0.01
	69.8%	94.6%	85.9%
log(BP)	0.41	0.28	0.27
	0.0%	0.0%	0.0%
FFINCP		−0.32	−0.29
		0.0%	0.0%

Note: Figures on top are time-series average values of regression coefficients; figures on bottom, the p-values of two-sided tests of the null hypothesis that the average value of the regression coefficient is zero.

Next we confirm A2. A2 is a model that adds FFINCP in place of IVOL. A2 finds a regression coefficient against FFINCP is -0.32 (p-value 0.0%), which is statistically significant and negative. This confirms that the long-term reversal effect measured with FFINCP cannot be explained by the FF3 Model.

[11] For the beta, this paper uses the regression coefficient against MKT estimated in the FF3 Model. All explanatory variables are normalized with overall data at each cross-section. The Tokyo Stock Exchange 33 sector classification is used as the sector dummy.

[12] Cross-sectional regressions using individual stock's return as the dependent variable generally caused heteroscedsaticity as smaller stocks have large error variance. It is known that this problem can be emprically mitigated by weighting the samples in proportion to the square root of their market capitalization.

Our focus then turns to A3. This model is added FFINCP to A1's independent variables. In A3 model, IVOL's regression coefficient is -0.05 (p-value 41.9%), which is negative but is not statistically significant even with a confidence level of 10%. However, the regression coefficient against FFINCP is -0.29 (p-value 0.0%), which is statistically significant and negative. The effect that remains even after adjusting for the long-term reversal effect can be considered as the pure idiosyncratic volatility effect. The result shows that the pure idiosyncratic effect (regression coefficient against IVOL in A3) is not statistically significant.

Our results indicate that a part of the long-term idiosyncratic volatility effect observed in Japanese equity markets is explained by the long-term reversal effect.

5.4 Interpretation of results

Below is the interpretation of the long-term volatility effect observed in Japanese equity markets in light of the findings of this paper.

(1) Large part of the total volatility effect can be explained by the idiosyncratic volatility effect.

(2) Stocks with high idiosyncratic volatility computed from past several years returns does not have high volatility over several years in the future, and idiosyncratic volatility tends to move like mean-reverting.

(3) Because of this, investing in stocks with high idiosyncratic volatility measured with prior return data will result in the holding of many stocks likely to experience declines in future volatility, and therefore realize low returns in the future.

(4) In addition, part of the idiosyncratic volatility effect stems from the long-term reversal effect.

(5) The idiosyncratic volatility effect adjusted for the long-term reversal effect is not statistically significant.

6. Conclusion

This paper analyzes long-term volatility in Japanese equity markets and finds that the main factor in the total volatility effect is the phenomenon that stocks with higher idiosyncratic volatility tend to have lower future returns (the idiosyncratic volatility effect). One factor explaining this effect is that stocks with high realized idiosyncratic volatility measured with data for several years in the past involve many stocks for which idiosyncratic volatility will decline in the future, and therefore tend to have lower future returns. One of the reasons why the idiosyncratic volatility effect is observed is because time-varying realized idiosyncratic volatility is used as a proxy for expected idiosyncratic volatility. Therefore, realized idiosyncratic volatility effect is likely not a puzzle that is inconsistent with the theory.

This paper also shows that realized idiosyncratic volatility unreasonably depends on the past returns and the idiosyncratic volatility effect includes an effect similar to the long-term reversal effect. When adjusted for the long-term reversal effect, the idiosyncratic volatility effect appears not to be large enough to have statistical significance.

The remainder of this paper comments on issues to be addressed in the future.

Recent years have seen a number of analyses of low return skewness effect that is similar to but different from long-term reversal effect. This paper performed an analysis of the idiosyncratic volatility effect adjusted for the long-term reversal effect, but it would be valuable to also include low return skewness effect.

This paper demonstrated that large part of the total volatility effect can be explained by the idiosyncratic volatility effect, but the idiosyncratic volatility effect is not the only cause of the total volatility effect. In Panel B of Exhibit 1, returns did not decline that much for stocks with low systematic volatility. This is also one factor in the total volatility effect. Fama and French [1992] report that high-B/M stocks have high average returns, and Table 2 of that paper contains average B/M for portfolios ranked by market beta. It finds that the lower the stock's beta, the higher the B/M. It would be interesting to perform a detailed analysis of the relationship between the value premium and the reasons why low-beta returns do not decline that much.

Finally, this paper uses volatility observed over the long period of 60 months and finds that realized idiosyncratic volatility may not be appropriate proxy for expected idiosyncratic volatility. Investigating which the volatility forecasting models are appropriate to estimate expected volatility would be useful to many practitioners who currently use realized volatility as a proxy for expected volatility.

This paper contains a basic analysis. It is hoped that its findings will prove useful in investment decision-making and quantitative analysis in the future.

The authors would like to thank anonymous referees. We received the very valuable suggestions and advice from them. Please acknowledge that the views expressed in this paper are those of the individual authors and do not necessarily reflect the position of the organization to which the authors is affiliated. Furthermore, any error that may be found in the work is that of the authors.

[References]

- Ishibe, M., Kakuda, Y., and Sakamaki, S. [2009] "Global Minimum Variance Portfolio and Volatility Effect", Securities Analysts Journal, 47 (12).(in Japanese)
- Kubota, K. and Takehara, H. [2007] " Re-examination of Effectiveness of Fama-French Factor Model", Modern Finance 22 (September). (in Japanese)
- Yamada, T. and Uesaki, I. [2009] "Low Volatility Strategy in Global Equity Markets", Securities Analysts Journal, 47 (6). (in Japanese)
- Yamada, T. and Nagawatari, M. [2010] " Investor Expectations and the Volatility Puzzle in the Japanese Stock Market", 48 (12). (in Japanese)
- Ang, A., R. Hodrick, Y. Xing, and X. Zhang [2009] "High Past Idiosyncratic Volatility and Low Future Returns: International and Further U. S. Evidence, " *Journal of Financial Economics,* 91, pp.1–23.
- Bali, T. G. and N. Cakici [2008] "Idiosyncratic Volatility and the Cross Section of Expected Returns, " *Journal of Financial and Quantitative Analysis,* 43, pp.29-58.
- Cao, X. and Y. Xu [2010] "Long-run Idiosyncratic Volatilities and Cross-sectional Stock Returns, " Working Paper.
- Fama, E. F. and K. French [1992] "The cross-section of expected stock returns, " *Journal of Finance*, 47, pp.427-465.
- Fama, E. F. and K. R. French [1993] "Common risk factors in the returns on stocks and bonds, " *Journal of Financial Economics*, 33, pp.3-56.
- Fu, Fangjian [2009] "Idiosyncratic Risk and the Cross-Section of Expected Stock Returns, " *Journal of Financial Economics,* 91, pp.24-37.
- Huang, W., Q. Liu, S. G. Rhee, and L. Zhang [2010] "Return Reversals, Idiosyncratic Risk, and Expected Returns, " *Review of Financial Studies*, 23, pp.147-168.
- Merton, R. [1987] "A Simple Model of Capital Market Equilibrium with

Incomplete Information, " *Journal of Finance,* 42, pp.483–510.

This work is an adaptation of a contribution made to the publication.