Low Volatility Strategy in Global Equity Markets

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Does a higher risk asset mean higher return? The authors investigate the relation between past volatility and future returns in Japanese and global equity markets. The results show that the low volatility anomaly covers a wide range of universes and that active management to reduce volatility can yield value added against a value-weighted portfolio.

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1. Introduction

1.1 Risk and Return

In modern portfolio theory (MPT), the efficient frontier is defined by a set of portfolios that yields maximum expected return for a given risk. With no risk-free asset, the efficient frontier is concave, therefore a higher risk portfolio should have higher expected return.

The capital asset pricing model (CAPM) holds that expected returns on individual assets and portfolios are proportional to the risk measured by beta to the market portfolio.

According to these theoretical models, expected return monotonically increases with risk. However, in reality, this relation is not so simple as earlier studies have discovered .

Ang et al. [2006] report that using a sample of U.S. stocks from 1963 to 2000, stocks with higher volatility, calculated by daily return for the past one month, produce significantly lower future returns which cannot be explained by CAPM or the three-factor model (Fama and French [1993]).

Looking at developed markets and computing volatility by weekly returns of the previous three years, Blitz and van Vliet [2007] find that higher volatility is associated with lower future returns. Also, they reveal that this result is unchanged for many sub-samples.

Bali and Hovakimian [2007] obtain the same result as Ang et al. [2006] and additionally, point out that implied volatility for a single stock in the U.S. market does not have predictive power for future returns.

In addition to these studies, many papers concerning the relation between idiosyncratic risks and future returns have been published as well. Idiosyncratic risk is risk that cannot be explained by common factors in asset pricing models (e.g. CAPM and FF model) and is sometimes termed residual risk.

Ang et al. [2006] also look into FF model based idiosyncratic risk and show it to have a negative correlation with future returns in the U.S. market. Extending this focus to developed markets, Ang et al. [2009] show the same results.

On the other hand, Malkiel and Xu [2006] and Malkiel [2008] indicate that higher volatility and idiosyncratic risk are connected with higher future returns, as opposed to the previous studies. It should be noted that they adopt in-sample datasets for the analysis.

Bali and Cakici [2008] suggest that the impact of idiosyncratic risks on future returns may vary depending on several parameters such as the frequency of return data to measure risk and the weighting of stocks in the portfolio to compare performance.

1.2 Minimum Variance Portfolio

The lowest risk portfolio on the efficient frontier is referred to as the global minimum variance portfolio, or simply, minimum variance portfolio (hereafter Min-Var). This portfolio is comprised of risky assets that minimize total risk.

Empirical studies show that Min-Var produces higher return while lower risk in comparison with a market capitalization weighted portfolio (hereafter MP).

Haugen and Baker [1991] report that over the period from 1972 to 1989 in the U.S. market, a low volatility portfolio outperformed MP. They also argue that it is unrealistic to hold assumptions required by CAPM, where MP stands as the most efficient portfolio.

Kleeberg [1995] shows that Min-Var exhibits superior performance to MP even for some markets: the U.S., Germany, the U.K., Japan, and Canada.

More recently, consistent results have been presented by Clarke [2006] for U.S. stocks from 1968 to 2005. Ishibe [2007] follows this sort of exploration for Japanese stocks from 1995 to 2007, after adjusting for value and momentum effects.

1.3 Methodology

As we describe above, the relationship between risk and return has been well investigated and documented from both an individual stock and portfolio basis. In general, analysis on the former basis has confirmed the existence of a low volatility anomaly and the latter that low volatility investments deliver value added, i.e. high risk-adjusted performance. This effectiveness of low volatility investments is assumed to be caused by the anomaly based on individual stocks.

In our paper, we attempt to investigate the relationship between past volatility and future returns applying a consistent methodology from the perspective of both an individual stock and portfolio basis.

It should be noted that here we assume the value added of low volatility investments is high risk-adjusted performance, where risk is defined by absolute risk not relative risk against some benchmark. Upon testing any anomalies, e.g. the value stock effect, in general, biases adjusted relative return or idiosyncratic component and their stability are tested as Ang et al. [2006] demonstrate.

Such tests, however, cannot evaluate the risk reduction effect that low volatility investments provide.As we show later, such tests are inappropriate for the evaluation of low volatility investments that tend to have low absolute risk but high relative risk.

Therefore, we use a mean return, an absolute risk, and the Sharpe ratio as performance measures and try to avoid the biases of a holding portfolio that interfere with the results.

In concrete, we investigate as follows:

First, for each universe, we construct portfolios by sorting stocks based on past volatility and test return differences between the lowest volatility portfolio and the highest one. It is the same as testing the performance provided by the portfolio that longs the lowest volatility portfolio and shorts the highest one.

Second, for each universe, we construct a Min-Var portfolio and compare its posterior performance with MP. Because many investors are restricted to short stocks, we assume short positions are not permitted. In addition, we test another low volatility strategy, which is designed to focus the anomaly on an individual stock basis.

1.4 Covariance Matrix

To construct Min-Var, a variance-covariance matrix of individual stock returns is required.

A sample covariance matrix is easily calculated with all past return series of individual stocks included in the universe. However, it is known that portfolio risk estimated by the sample covariance may incur large estimation errors unless the number of observations sufficiently exceeds the size of the sample. Since the universe we target consists of several thousand stocks at most, several hundred years return observations on a monthly basis are required at least.

To avoid this problem, we use the Bayesian Shrinkage method which has the advantage that it is not necessary to assume a specific market structure.

The basic idea is to shrink the covariance matrix to a prior matrix that can be estimated by some assumption. In concrete, estimated covariance is defined by a weighted average of the sample covariance S and the prior matrix F:

$$\alpha F + (1-\alpha)S$$

where α is a shrinkage coefficient and increases when the uncertainty of S increases.

We assert a prior estimation matrix that is composed of identical variances and identical pairwise correlation coefficients proposed by Frost and Savarino [1986]. Referring to Ledoit and Wolf (2004), an optimal $\hat{\alpha}$ is estimated by:

$$\hat{\alpha} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}\left\{(y_{it}-\bar{y}_i)(y_{jt}-\bar{y}_j)-s_{ij}\right\}^2\right]}{T\cdot\sum_{i=1}^{N}\sum_{j=1}^{N}\left(f_{ij}-s_{ij}\right)^2}$$

where T is the number of observations, N is the number of stocks in the universe, y_{it} is return on stock i in month t, $\bar{y}_i$ is average return on stock i, and s_{ij}, f_{ij} are components of S and F. We set maximum value of $\hat{a}$ as 0.9[1].

2. Data and Methodology

2.1 Universe

Our sample universe consists of stocks by market, region, and sector, as well as quintile portfolios sorted by size, price to book value, and price momentum. As for each sector and quintile portfolio, we adopt stocks listed on the 1st Section of the Tokyo Stock Exchange for Japanese equities and MSCI and FTSE constituents in developed markets for global equities.

Market segments, regions, and sectors, respective start dates, and average numbers of stocks are summarized in Table 1. Global equities are divided into six developed and three emerging regions. Regarding sector classification, we employ the GICS 10 sectors for global equities and seven based on the SICC (Securities Identification Code Committee) classification for Japanese equities[2].

2.2 Data

Our analysis is from January 1990 to October 2008, using MSCI and FTSE constituents at each time point. As for Japanese stocks, a longer period is taken, from January 1975 to October 2008. We exclude

[1] The assumptions of Ledoit and Wolf (2004) (LW, hereafter) are different from ours on the following three points: (1) LW assumes each prior estimated variance as each sample variance, (2) the covariance terms between the estimated errors of S and F (we ignore them) is considered, (3) the upper limit of $\hat{a}$ is 1. Clarke, de Silva and Thorley (2006) provide a similar optimal coefficient. Their assumptions regarding (1) and (2) are the same as ours and they report that the optimal α does not exceed 1 in their conditions. One feature of the Bayesian approach is to determine such prior estimates based on subjective decisions. According to our analysis (not reported), F assumed by LW provides low robustness against outliers in the sample variances, and the covariance term of (2) has only small effects on the results. Optimal α occasionally touches the upper limit of 1 when the market drastically fluctuates, e.g. when the Lehman Brothers shock happened. If $\hat{a}$ reaches the upper limit, our F forces Min-Var to be an equally-weighted portfolio. Frost and Savarino (1986) also assume as we do regarding (1) and (2), however, they focus on cases when the number of observations substantially exceeds the number of assets.

[2] The seven sectors are as follows (SICC classification in parenthesis): Materials (Textiles & Apparel, Pulp & Paper, Chemicals, Glass & Ceramics Products, Iron & Steel, and Nonferrous Metals), Manufacturing (Machinery, Electric Appliances, Transportation Equipment, and Precision Instruments), Miscellaneous Manufacturing (Foods, Pharmaceuticals, Oil & Coal Products, and Other Products), Transportation & Utilities (Electric Power & Gas, Land Transportation, Marine Transportation, Air Transportation, Warehousing & Harbor Transportation Services, and Information & Communication), Services (Wholesale Trades, Retail Trade, and Services), Financial (Banks, Securities, Insurance, and Other Financials), and Miscellaneous Non-Manufacturing (Fishery, Agriculture & Forestry, Mining, Construction, and Real Estate).

REITs and some real estate-related stocks from our universe due to their bond-like characteristics since they differ from others in terms of risk-return features.

Stock returns are measured monthly, with dividends in local currency terms and the relevant risk-free rate deducted.

Data provided by FactSet is used to calculate stock market capitalization and price to book value with a certain time lag reflecting availability of financial statement data. We alternatively utilize a variety of data on Japanese stocks provided by Integrated Data Services of Nomura Research Institute.

In this paper we define volatility as standard deviation of monthly returns over the last 60 months, requiring more than 12 monthly returns. Furthermore, momentum of a stock is prescribed by the average 12 months' log returns[3].

The performance of quantile portfolios is measured as the simple average of return of stocks in the portfolio if composed of more than five stocks.

A sample covariance matrix is derived from the last 60 months' return as long as return for the next month and overall period can be observed when the universe includes more than ten stocks.

2.3 Performance Measurement and Evaluation

Portfolio performance statistics are grounded in geometric returns. In our study, we evaluate performance difference between two portfolios in terms of return, total risk, and Sharpe ratio[4].

For performance evaluation, the Welch method assesses return difference due to difficulty in assuming homoscedasticity.

Next, the standard error of the difference between two Sharpe ratios is expressed as follows:

$$SE = \sqrt{\frac{1}{T}\left[2(1-\rho_{1,2}) + \frac{1}{2}\left(SR_1^2 + SR_2^2 - SR_1 \cdot SR_2 \cdot \rho_{1,2}^2\right)\right]}$$

where T is number of observations, SR_i is portfolio i'th Sharpe ratio, and $\rho_{1,2}$ denotes the correlation of portfolio returns.

The two side p-value of the null hypothesis $SR_1 = SR_2$ is described as:

$$2\Phi\left(-\frac{|SR_1 - SR_2|}{SE}\right)$$

where $\Phi(\cdot)$ is defined as the cumulative distribution function of standard normal distribution.

3 Some studies drop the latest month's return for momentum, but we do not.

4 Although the information ratio (the ratio of return difference over relative risk) is widely accepted, we do not adopt it because the low volatility strategy in general does not intend to earn relative return by taking relative risk.

3. Results

3.1 Quintile Portfolio

We constructed quintile portfolios with equal names by ranking stocks based on individual past volatility at every month end for both Japanese equity (listed on Tokyo Stock Exchange 1st Section [TSE1]) and global equity (developed countries) universes. The quintiles and universe portfolio for comparison are equally weighted.

The portfolio attributions of quintile 1 (Q1) with lowest volatility stocks are shown in Table 2, which illustrates the difference between Q1 and the universe over the whole period. As for Japanese equity sectors, Q1 tends to overweight Financials and Utilities and underweight Manufacturing. As for global equities, Q1 overweights Utilities, Consumer Staples, and Financials in the U.S. while underweights Consumer Discretionary and Information Technology in Japan. Regarding 'Attribution', there is a slightly bias on 'Large'. As the minimum and maximum exposures indicate, these tendencies are not stable.

Table 3 shows the statistics of next month returns for each quintile portfolio. Comparing Q1 and Q5, Q5 (the highest volatility) tends to deliver higher risk, lower return, and a lower Sharpe ratio. As for the difference between Q1 and Q5, the return difference is not statistically significant but differences in risk and the Sharpe ratio are significant at the 5% level.

Table 4 gives time series regression results over the whole period between Q1 and the universe. In both markets, betas are approximately 0.6 and alphas more than 3% per annum, which are statistically significant at the 5% level.

Sub-universe statistics for Japanese and global equities are shown in Table 5 Panel A and B, respectively. We omit the results except for statistics of the differences between Q1 and Q5. As for Sharpe ratios, all figures are positive and most statistically significant, which means that the low volatility anomaly is robust to changes in country, sector, and other attributions.

Table 6 represents the statistics calculated during several periods of five or ten years for Japanese and global equities. As the high volatility portfolio Q5 occasionally outperforms Q1, we do not consistently observe significant differences.

To sum up, we can confirm the existence of a broad and robust low volatility anomaly from a long-term viewpoint.

3.2 Minimum Variance Portfolio

At every month-end we estimated the covariance matrix by the methodology described above and constructed minimum variance portfolios. Table 7 shows next month returns of Min-Var and MP for

Japanese and global universes. The average number of Min-Var stocks is 142 for Japan and 245 for global equities, and average optimal shrinkage coefficient 0.50 for Japan and 0.58 for global equities.

Min-Var does not have statistically significant differences vs. MP in terms of mean return and Sharpe ratio but delivers significantly lower risk. Relative risk against MP (tracking error) is rather higher than figures for general active management, which are typically supposed to be between 2% and 5% per annum.

As for sub-universes in Japanese equities, Table 8 Panel A summarizes differences and ratios between Min-Var and MP. Min-Var's risk is approximately 70% of MP's and differences are statistically significant for most universes. Differences in mean returns are not significant and Min-Var tends to deliver a higher Sharpe ratio but cases where the difference is statistically significant are few.

Table 8 Panel B shows sub-universe results for global equities. Comparing Panel A and B, the results are similar with Min-Var tending to provide higher results for global equities. Among regions, Japan (Panel B)'s Min-Var Sharpe ratio is lower than MP's, which is not statistically significant.

In Japanese equities, the average return of Min-Var decreases in value quintiles, which means that attributions of low volatility portfolios have something of a relation with value attribution. However, it is not the case in global equity markets.

Table 9 represents performance comparison results during five to ten years for Japanese and global equities. In the long run, return difference between Min-Var and MP is large, which means comparison results vary somewhat depending on observation periods.

The papers we introduced earlier report that Min-Var delivers higher returns than MP, however they do not say whether the result is statistically significant or not. Our analysis shows that volatility of Min-Var is significantly lower, however, mean return and Sharpe ratio are generally higher but differences are not significant. Since the concept of the Minimum Variance Portfolio has a long history dating back to MPT and is not introduced with the intention of outperforming other portfolios, we regard our results as reasonable. In other words, Min-Var delivers a comparable return to MP regardless of significantly lower risk, which means Min-Var is not just a low beta portfolio.

3.3 1/Var Weighted Portfolio

From the analysis of quintile portfolios above, we observe statistically significant low volatility anomalies on an individual stock basis. On a portfolio basis for Min-Var, volatility decreases significantly but there are no significant return differences. To absorb the anomalies on an individual stock basis into a portfolio construction scheme efficiently, a one-over-variance portfolio (1/Var, hereafter), which determines each holding weight proportional to the inverse of a sample variance, can be introduced. We can recognize 1/Var as one version of Min-Var that is constructed with a covariance whose non-diagonal components are set to be zero.

Table 10 shows comparison results among 1/Var, Min-Var, and MP. Volatility of 1/Var is 90% of MP and higher than Min-Var. Average return of 1/Var is higher than those of MP and Min-Var but differences are not statistically significant. The Sharpe ratio of 1/Var is significantly higher than that of MP for both universes.

This result shows robust low volatility anomalies on an individual stock basis. On a portfolio basis, robustness of the effect is supposed to be weaker because the non-diagonal components of the covariance matrix we use here have large estimation errors[5].

4. Discussion and Consideration

4.1 Low Volatility Anomaly

There are four possible reasons to explain the low volatility anomaly found in most markets.

(i) False recognition of future fundamentals by investors and analysts

When a company expands into a new but riskier business, it also expects to gain compensation profits for taking on the corresponding uncertainty. Investors and analysts recognize such uncertainty and, as a consequence, their caution vis-à-vis the company will intensify (Athanassakos and Kalimipalli [2003]). Nevertheless, in return they will still expect appropriate return. If a high risk business delivers less profit than expected, the stock price would fall due to the excessive expectations originally anticipated.

(ii) Selling pressure from investment policy

Some investors may have investment guidelines such as realizing a profit and cutting their losses whenever a stock price moves above or below a limit. Such policy will have a greater impact on high volatility stocks.

(iii) Overexpectation for high beta stocks on the part of overly-optimistic investors

In a situation where most investors believe CAPM holds in reality and simultaneously have an overly-optimistic view of market prospects, they act on higher beta stocks (the higher volatility stocks in many cases) considering them overvalued since such stocks are expected to perform better than the market.

(iv) Fallacy stemming from investment managers' business decision

Inflows from investors often seem to be biased toward assets, managers, and time which have a good record. As Blitz and van Vliet [2007] point out, if investment managers maximize their business benefit, they have a strong incentive to operate in a booming market with gearing policy in hope of money inflows. As a result, riskier stocks tend to be preferred beyond the reality.

[5] Our analysis (not reported) shows that the performance of Min-Var can be improved by using a covariance matrix estimated by a well designed multi-factor model.

Although some of these reasons are touched on in previous studies, none specify the exact cause. Probably, the dominant reason may vary with the market and time—at the end of the day, more than one reason induces the low volatility anomaly. It is hoped further studies will uncover the reasons for this finding.

4.2 Minimum Variance Portfolio

For equity investment, MP stands as the dominant position in practice. That is, most passive investments aim to strictly track MP, and most active investments should be based on MP to some extent. The theoretical underpinning of this framework comes from CAPM, which says that the market portfolio is the most efficient in terms of risk-adjusted return, though many unrealistic assumptions are required.

Min-Var, as well as MP, is located on the efficient frontier and exhibits an as good or better ex-post Sharpe ratio according to results in this paper.

Not only for equity investment, but for asset allocation MP is used as the benchmark to define risk premium for equities. If we substitute Min-Var for MP, it would be possible to enjoy higher return with lower risk. Or, if we have a total risk budget, it is possible to allocate more to equity by Min-Var, which may contribute to higher return for the overall portfolio.

At first glance, the portfolio constructions of MP and Min-Var conflict because the former is just based on the portion held in the market and the latter reflects risk-return efficiency. However, MP and Min-Var do not compete with each other. Rather, both sides complement each other aiming at higher efficiency through different approaches to the method and concept of portfolio construction.

5. Conclusion

We conducted empirical tests regarding the relations between past volatility and ex-post returns, risks, Sharpe ratios in Japanese and global equity markets and sub-universes divided by sector, attributions such as company size, and periods. These tests can be divided into two parts.

First, we construct quintile portfolios by ranking individual stocks by past volatility and measure next month returns for each quintile. We find that the lowest volatility portfolio delivers a statistically significantly higher Sharpe ratio than the highest volatility portfolio for most universes.

Second, we compare the performance of Min-Var with MP. According to our test, Min-Var generally produces a higher return and a higher Sharpe ratio than MP but differences are not statistically significant, while Min-Var delivers a significantly lower risk. 1/Var, which is a portfolio construction scheme that focuses the anomaly on an individual stock basis, produces a significantly higher Sharpe ratio than MP.

These results show that broad low volatility anomalies exist in global equity markets and that low volatility strategies can be expected to produce a comparable or higher return than MP at a lower portfolio volatility in the long run.

For investors who are highly risk-averse and/or have a long-term investment horizon, to invest in low volatility strategies is a rational choice when aiming for more efficient performance.

We thank the referees for providing constructive comments and help in improving the content of this paper.

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Table 1 Universe Statistics

	Start date	Number of stocks
Japan Equity		
Market		
TSE1	Jan-75	1,222
non-TSE1	Jan-75	1,313
Sector (TSE1)		
Material	Jan-75	248
Manf	Jan-75	282
Misc Manf	Jan-75	164
Trans & Utilities	Jan-75	87
Services	Jan-75	181
Financials	Jan-75	128
Misc Non-Manf	Jan-75	131
Global Equity		
Market		
Developed	Jan-90	2,069
Emerging	Dec-01	936
Region		
US	Jan-90	593
Canada	Jan-91	108
UK	Jan-90	191
Europe excl. UK	Jan-90	551
(Japan)	Jan-90	448
AP excl. Japan	Jan-90	185
EM Asia	Dec-01	547
EM EMEA	Dec-01	240
EM LATAM	Dec-01	150
Sector (Developed countries)		
Energy	Jan-90	83
Materials	Jan-90	244
Industrials	Jan-90	400
Cons. Disc.	Jan-90	371
Cons. Staples	Jan-90	180
Health Care	Jan-90	111
Financials	Jan-90	349
IT	Jan-90	170
Telecom services	Jan-90	49
Utilities	Jan-90	106

Notes

1. ”Global Equity” is comprised of constituents MSCI and FTSE index.
2. (Japan), as one of regional segment in “Global Equity”, is different from ”Japan Equity”.
3. REITs and some of real estate related stocks are excluded.

Table 2 Attributions of Low Volatility Portfolio

	Portfolio exposure		
	Mean	Min	Max
Japan Equity (TSE1)			
Sector			
Material	−8.0	−18.6	7.9
Manf	−12.6	−20.9	1.2
Misc Manf	2.9	−8.0	11.1
Trans & Utilities	5.2	−6.1	14.3
Services	0.0	−6.5	9.6
Financials	16.9	2.4	34.9
Misc Non-Manf	−4.3	−8.9	4.4
Attribution			
Size	0.4	0.0	0.8
Value	0.3	−0.6	0.7
Momentum	0.1	−0.8	0.9
Global Equity (Developed Countries)			
Region			
US	7.8	−9.6	21.5
Canada	3.0	−0.2	9.6
UK	1.5	−1.9	8.0
Europe excl. UK	−3.1	−15.7	3.6
(Japan)	−10.4	−20.8	−1.3
AP excl. Japan	1.2	−7.4	9.4
Sector			
Energy	0.9	−4.1	2.9
Materials	−3.7	−8.4	0.9
Industrials	−7.1	−13.3	−2.2
Cons. Disc.	−8.8	−11.2	−6.1
Cons. Staples	7.8	3.6	10.9
Health Care	0.0	−3.6	3.0
Financials	6.1	0.6	13.4
IT	−7.3	−12.2	−2.4
Telecom services	0.7	−2.5	3.4
Utilities	11.4	8.7	15.5
Attribution			
Size	0.4	0.0	0.6
Value	−0.1	−0.5	0.1
Momentum	0.2	−0.6	0.9

Notes For region and sector segments, numbers are the time-series average of the weight in the low volatility portfolio relative to its universe. For attribution quintiles, numbers are the time-series average of Z-score in the low volatility portfolio relative to its universe. Z-score is calculated by standardizing ranking of the attribution of stocks in the universe. Positive numbers for Size, Value and Momentum mean large, undervalued (high B/P) and high return, respectively.

Table 3 Performance of Quintile Portfolios

(p.a.)	Universe	Low Q1	Quintiles 2	3	4	High Q5	Q1−Q5
Japan Equity (TSE1)							
Return (%)	3.38	5.46	5.08	4.33	3.17	−2.38	7.84
Risk (%)	19.7	14.0	17.8	20.3	22.6	27.0	51.8 **
Sharpe ratio	0.17	0.39	0.29	0.21	0.14	−0.09	0.48 **
Global Equity (Developed countries)							
Return (%)	0.94	3.77	2.19	0.97	−0.02	−3.51	7.28
Risk (%)	16.7	11.2	14.0	16.6	19.4	26.2	42.7 **
Sharpe ratio	0.06	0.34	0.16	0.06	0.00	−0.13	0.47 **

Notes 1. "Q1-Q5" means the difference for Return and Sharpe Ratio and the ratio for Risk.

2. "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 4 Performance Test Results of the Lowest Volatility Quintile

	alpha (% p.a.)	beta	R−square (%)
Japan Equity (TSE1)	3.29 **	0.64 **	81.2
Global Equity (Developed)	3.21 **	0.60 **	79.4

Notes "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 5 Performance of Quintile Portfolios in Various Sub-Universes

Panel A Japan Equity	Q1-Q5 or Q1/Q5		
(p.a.)	Return (%)	Risk (%)	Sharpe ratio
Market Section			
TSE1	7.8	51.8 **	0.48 **
non-TSE1	3.8	45.0 **	0.34 **
Sector			
Material	6.2	58.6 **	0.32 **
Manf	6.2	62.1 **	0.35 **
Misc Manf	8.6 *	61.6 **	0.51 **
Trans & Utilities	10.3 *	57.2 **	0.52 **
Services	6.2	59.2 **	0.32 **
Financial	2.8	44.4 **	0.15
Misc Non-Manf	13.6 **	59.1 **	0.51 **
Size Quintiles			
1 Small	8.1	59.8 **	0.45 **
2	10.1 *	59.6 **	0.48 **
3	10.4 **	57.0 **	0.54 **
4	7.3	57.2 **	0.39 **
5 Large	8.4 *	61.3 **	0.50 **
Value Quintiles			
1 High B/P	0.0	56.6 **	0.21 *
2	2.1	55.8 **	0.24 **
3	3.9	52.4 **	0.28 **
4	5.1	55.3 **	0.24 *
5 Low B/P	10.8 *	57.4 **	0.41 **
Momentum Quintiles			
1 Low Return	11.6 *	62.8 **	0.50 **
2	5.6	64.5 **	0.32 **
3	7.3	59.5 **	0.45 **
4	8.8 *	61.7 **	0.51 **
5 High Return	8.9 *	63.1 **	0.45 **

Panel B Global Equity	Q1-Q5 or Q1/Q5		
(p.a.)	Return (%)	Risk (%)	Sharpe ratio
Market			
Developed	7.3	42.7 **	0.47 **
Emerging	11.6	43.7 **	0.77 **
Region			
US	2.3	38.7 **	0.26
Canada	17.2 **	37.2 **	0.99 **
UK	7.1	46.6 **	0.30 *
Europe excl. UK	7.1	48.6 **	0.41 **
(Japan)	7.6	52.2 **	0.14
AP excl. Japan	10.9	39.1 **	0.64 **
EM Asia	15.2	47.6 **	0.83 **
EM EMEA	8.1	40.8 **	0.54 *
EM LATAM	11.3	45.2 **	0.75 **
Sector			
Energy	3.2	51.4 **	0.26
Materials	2.4	60.8 **	0.16
Industrials	6.9	48.0 **	0.34 **
Cons. Disc.	3.7	58.6 **	0.15
Cons. Staples	6.3	57.3 **	0.60 **
Health Care	5.2	58.4 **	0.43 **
Financials	2.3	54.6 **	0.19
IT	3.0	45.3 **	0.14
Telecom services	9.7	37.4 **	0.41 *
Utilities	1.3	47.8 **	0.30
Size Quintiles			
1 Small	3.3	41.8 **	0.25
2	6.2	41.9 **	0.39 **
3	10.2	43.8 **	0.54 **
4	7.9	43.7 **	0.48 **
5 Large	7.8	43.1 **	0.45 **
Value Quintiles			
1 High B/P	2.7	43.9 **	0.22
2	3.3	49.6 **	0.35 **
3	7.5	45.0 **	0.46 **
4	9.1	46.0 **	0.48 **
5 Low B/P	12.2 *	37.9 **	0.62 **
Momentum Quintiles			
1 Low Return	9.9	57.2 **	0.29 **
2	8.0	60.4 **	0.36 **
3	5.6	56.0 **	0.37 **
4	6.2	52.0 **	0.57 **
5 High Return	1.7	53.1 **	0.33 *

Notes 1 . "(Japan)" in Panel B is different from Panel A on the sample period.

2. "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 6 Performance of Quintile Portfolios in Various Periods

(p.a.)	Return (%) Q1 L	Return (%) Q5 H	Return (%) Q1-Q5	Risk (%) Q1 L	Risk (%) Q5 H	Risk (%) Q1/Q5	Sharpe ratio Q1 L	Sharpe ratio Q5 H	Sharpe ratio Q1-Q5
Japan Equity (TSE1)									
1970s	8.3	8.3	0.0	6.4	16.2	39.7 **	1.29	0.51	0.78 *
1980s	20.5	10.4	10.2 *	12.2	14.2	86.1	1.68	0.73	0.95 **
1990s	-9.8	-15.2	5.4	18.3	38.1	48.2 **	-0.54	-0.40	-0.14
2000s	4.1	-8.4	12.5	11.7	27.5	42.6 **	0.35	-0.30	0.65 **
Global Equity (Developed countries)									
1990-1994	-0.4	-1.3	0.9	11.1	19.9	56.0 **	-0.04	-0.07	0.03
1995-1999	10.6	13.7	-3.2	9.5	19.2	49.8 **	1.11	0.72	0.39
2000-2004	6.6	-10.8	17.4	9.2	31.6	29.2 **	0.71	-0.34	1.06 **
2005-2008	-3.4	-19.4	16.0	14.8	32.6	45.4 **	-0.23	-0.60	0.37 *

Note: "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 7 Performance of Min-Var

(p.a.)	MP A	Min-Var B	Min-Var B-A
Japan Equity (TSE1)			
Return (%)	0.88	0.83	-0.06
Risk (%)	17.7	11.7	65.8 **
Sharpe ratio	0.05	0.07	0.02
Tracking error (%)		11.9	
Global Equity (Developed countries)			
Return (%)	0.40	1.79	1.38
Risk (%)	15.4	11.2	72.7 **
Sharpe ratio	0.03	0.16	0.13
Tracking error (%)		8.2	

Notes: 1, "B-A" means the difference for Return and Sharpe Ratio, the ratio for Risk.

2. "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 8 Performance of Min-Var in Various Sub-Universes

Panel A Japan Equity

(p.a.)	Min-Var(B) vs. MP(A) Return B-A (%)	Risk B/A (%)	Sharpe ratio B-A
Market Section			
TSE1	-0.06	65.8 **	0.02
non-TSE1	1.24	56.9 **	0.18
Sector			
Materials	0.99	79.5 **	0.07
Manf	1.83	78.0 **	0.13
Misc Manf	-0.40	92.2	-0.01
Trans & Utilities	3.20	85.0 **	0.21 *
Services	1.16	73.8 **	0.08
Financials	1.34	55.6 **	0.08
Misc Non-Manf	4.11	78.8 **	0.22 **
Size Quintiles			
1 Small	-1.10	73.5 **	0.05
2	-1.71	68.4 **	-0.05
3	-0.17	64.0 **	0.05
4	-1.11	71.6 **	-0.05
5 Large	0.12	72.1 **	0.02
Value Quintiles			
1 High B/P	-0.56	78.3 **	0.09
2	-3.56	70.9 **	-0.11
3	-3.12	72.3 **	-0.14
4	-0.41	71.7 **	-0.04
5 Low B/P	0.96	70.4 **	-0.03
Momentum Quintiles			
1 Low	3.76	77.8 **	0.21 **
2	2.32	79.0 **	0.16 *
3	1.46	73.9 **	0.14
4	0.81	74.4 **	0.08
5 High	-1.79	73.8 **	-0.10

Panel B Global Equity

(p.a.)	Min-Var(B) vs. MP(A) Return B-A (%)	Risk B/A (%)	Sharpe ratio B-A
Market			
Developed	1.38	73.5 **	0.14
Emerging	6.93	56.6 **	0.63 **
Region			
US	-0.41	79.1 **	0.03
Canada	1.28	71.3 **	0.21
UK	0.71	88.1 **	0.05
Europe excl. UK	3.45	68.3 **	0.32 **
(Japan)	-0.91	74.9 **	-0.17
AP excl. Japan	0.43	74.3 **	0.15
EM Asia	6.69	58.4 **	0.62 **
EM EMEA	9.13	63.2 **	0.60 **
EM LATAM	7.24	64.3 **	0.55 **
Sector			
Energy	-0.30	81.3 **	0.04
Materials	1.25	77.9 **	0.07
Industrials	1.78	79.1 **	0.11
Cons. Disc.	0.92	80.6 **	0.05
Cons. Staples	0.16	81.7 **	0.08
Health Care	-0.89	88.0 **	-0.03
Financials	3.31	67.1 **	0.23 *
IT	2.85	70.4 **	0.18
Telecom services	2.09	83.3 **	0.15
Utilities	2.02	78.8 **	0.23 *
Size Quintiles			
1 Small	1.99	72.6 **	0.21 **
2	-0.55	72.9 **	0.00
3	0.32	73.7 **	0.07
4	-0.33	75.4 **	0.00
5 Large	0.27	73.3 **	0.03
Value Quintiles			
1 High B/P	0.68	70.6 **	0.09
2	0.65	69.0 **	0.14
3	0.82	73.5 **	0.12
4	1.25	76.1 **	0.11
5 Low B/P	-0.29	79.0 **	-0.04
Momentum Quintiles			
1 Low Return	2.59	69.8 **	0.08
2	2.74	78.7 **	0.14
3	3.03	81.9 **	0.24 **
4	1.34	83.1 **	0.15
5 High Return	-0.21	75.0 **	0.07

Notes 1. "(Japan)" in Panel B is different from Panel A on the sample period.

2. "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 9 Performance of Min-Var in Various Periods

	Return (%)			Risk (%)			Sharpe ratio		
	MP	Min-Var		MP	Min-Var		MP	Min-Var	
(p.a.)	A	B	B-A	A	B	B/A	A	B	B-A
Japan Equity (TSE1)									
1970s	5.5	9.6	4.2	10.7	7.1	66.2 **	0.51	1.37	0.85 **
1980s	14.7	15.1	0.4	14.5	9.7	66.8 **	1.01	1.56	0.55 *
1990s	−7.9	−15.0	−7.0	22.7	14.6	64.3 **	−0.35	−1.03	−0.68 **
2000s	−5.6	0.4	5.9	17.2	10.1	58.6 **	−0.32	0.04	0.36
Global Equity (Developed countries)									
1990-1994	−2.6	−1.4	1.2	13.8	13.7	99.1	−0.19	−0.10	0.09
1995-1999	15.4	7.4	−8.1	13.2	9.5	71.9 **	1.17	0.77	−0.39
2000-2004	−4.9	3.4	8.2	15.1	8.2	54.4 **	−0.32	0.41	0.73 **
2005-2008	−8.4	−3.5	4.9	19.2	12.9	67.2 **	−0.44	−0.27	0.17

Note: "*", "**" means statistically significant at the 10%, the 5%.level, respectively.

Table 10 Performance of Min-Var and 1/Var

	MP	Min-Var		1/Var	
(p.a.)	A	B	B-A	C	C-A
Japan Equity (TSE1)					
Return (%)	0.88	0.83	−0.06	4.70	3.82
Risk (%)	17.7	11.7	65.8 **	16.8	95.1
Sharpe ratio	0.05	0.07	0.02	0.28	0.23 **
Global Equity (Developed countries)					
Return (%)	0.40	1.79	1.38	2.53	2.12
Risk (%)	15.4	11.2	72.7 **	13.6	88.8 **
Sharpe ratio	0.03	0.16	0.13	0.19	0.16 **

Notes 1 . "(Japan)" in Panel B is different from Panel A on the sample period.

2. "*", "**" means statistically significant at the 10%, the 5%.level, respectively.